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The second level is a review of suitability for publication, implemented on the paper by two referees, selected within the Editorial Board, the Scientific Committee or externally among academics, scholars, experts on the subject who assess the content and the form.

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The papers shall be presented in Microsoft Word format, font Times New Roman 10 and shall have between 5.000 and 12.000 words; tables and graphs are welcome.

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First and second generation lookback and barrier options: enhancing pricing accuracy through Conditional Monte Carlo

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Abstract

This paper addresses the challenges associated with pricing exotic options, specifically path-dependent ones, with a focus on the limitations of standard Monte Carlo simulations and the advantages provided by Conditional Monte Carlo methods, introduced by Babsiri and Noel in 1998. Path dependent options, such as first and second-generation barrier and lookback options, require continuous monitoring of asset prices throughout their lifetime, making accurate pricing computationally demanding and prone to errors when using traditional Monte Carlo methods.
This work begins by presenting different exotic options, offering a detailed comparison between the exact pricing formulas and the results obtained from Crude Monte Carlo simulations. The Conditional Monte Carlo method is then applied to address the bias introduced by discrete monitoring intervals in the simulations, a critical issue in path-dependent options. A market case based on the valuation of a Bonus Cap certificate has also been shown.

Key Words: Exotic options, Path-dependent options, continuous monitoring, Brownian Bridge, Conditional Monte Carlo, Barrier Options, Lookback Options, Bonus Cap certificate.

JEL codes: C53, C63, G12, G32

1) Introduction

The valuation of many complex financial instruments, for which there are no analytical pricing formulas, is done through the Monte Carlo technique, which involves the integration of the stochastic differential equation governing the dynamics of the underlying, with the aim of deriving the financial variables that constitute the pay-off of the derivative (Giribone, 2024).
Generally, such an approach, called Crude Monte Carlo, does not introduce any numerical errors that would be critical enough to compromise weak convergence to the fair-value of the financial instrument (Giribone and Ligato, 2012).
However, there is a category of path-dependent options that exhibit significant divergence from the expected value, so that the reliability of the approach is invalidated (Tropiano, 2024).
To reduce the error committed in critical cases to reach acceptable values, the literature proposes the Conditional Monte Carlo variant, which is based on the probabilistic method known in literature as Brownian Bridge (Huynh, Lai and Soumarè, 2008).
In fact, this arrangement allows to drastically reduce the discretization error introduced by classical stochastic integration in first- and second-generation barrier and lookback options, which involve continuous monitoring of the underlying asset (Babsiri, Noel, 1998).
Such a simulation bias correction is therefore crucial to reach an accurate estimation of the fair-value of any derivative whose payoff depends on the extreme values reached by the underlying equity during the life of the contract.
The valuation of financial options has long been a cornerstone of quantitative finance, with particular interest in the complex pricing of exotic options like barrier and lookback options.
Barrier options are one of the most common types of exotic option, in which the payoff depends on whether the underlying asset price reaches a certain barrier level. Various methodologies have been proposed to address the challenges in pricing these instruments.
Reiner and Rubinstein (1991) provided a closed formula for the valuation of the standard type of Barrier options. Carr (1995) introduced two modifications to the valuation of barrier options: the first allows for an initial protection period during which the option cannot be knocked out, while the second considers an option which is only knocked out if a second asset reaches an upper barrier.
Other notable works include Metwally and Atiya (2003), who developed fast Monte Carlo methods for pricing barrier options in jump diffusion processes. This approach became a crucial tool for a more accurate and computationally efficient pricing of options. In a similar vein, Wang et al. (2009) proposed a hybrid approach combining binomial models and Monte Carlo simulations, which allowed for flexibility in modeling complex boundary conditions, further enhancing the accuracy of barrier option pricing.
Additionally, Sudding and Kalla (2021) introduced a method combining Monte Carlo simulations and binomial lattice models to estimate the price of lookback options, demonstrating the utility of lattice-based methods in the valuation of barrier options as well. Their approach allows for precise computation while maintaining reasonable computational complexity, a common challenge in the field.
Geman and Yor (1996) presented a probabilistic approach for pricing and hedging double-barrier options, considering a continuous-time framework.
Ikeda and Kunitomo (1992) studied the valuation of the second generation type double-barrier option, in which both a Lower barrier and an Upper barrier are present; while another second-generation type of the instrument, the soft-barrier option, has been valued with a closed formula from Hart and Ross (1994).
Lookback options, which allow the holder to "look back" at the underlying asset price during the life of the option and base the payoff on either the maximum or the minimum price reached, have drawn significant attention in recent years. Pricing these options is particularly challenging due to the need to track the path of the underlying asset over time, requiring advanced numerical methods.
As previously said, Sudding and Kalla implemented binomial lattice models along Monte Carlo simulations to estimate their price; Singirankabo (2020) addressed the pricing of lookback options using multinomial lattice methods, offering a computationally efficient

solution for these path-dependent options. Their method incorporates both the dynamics of the asset price and the boundary conditions necessary for accurate pricing. This approach, though rooted in classical lattice models, provides a modern solution to a problem that has historically been treated with more computationally demanding methods.
The work by Kudryavtsev et al. (2024) extended the Monte Carlo approach for pricing lookback options under Lévy processes, offering a comprehensive solution to pricing these instruments in markets with jumps and stochastic volatility. This work represents a step toward understanding the behavior of lookback options in more realistic financial models, especially those incorporating heavy tails and discontinuities in asset prices.
Further contributions include the study by Chen et al. (2019), which explored the pricing of lookback options using mixed fractional Brownian motion. Their research provides insights into the pricing of lookback options in markets with long memory, a phenomenon that is increasingly recognized as important in the modeling of financial markets.
Numerous studies have also explored hybrid methods and approximation techniques to improve the efficiency and accuracy of option pricing models. Babbs (2000) examined the binomial valuation of lookback options, proposing an approximation that simplifies the path-dependent nature of these options while maintaining reasonable accuracy. Grosse-Erdmann and Heuwelyckx (2016) further investigated binomial approximations for lookback options, focusing on improving the numerical stability and convergence properties of these methods.
In a more recent approach, Febrianti (2022) applied adaptive differential evolution methods with learning parameters to approximate the pricing of barrier options. This adaptive technique allows for fine-tuning of the parameters during the optimization process, leading to more accurate pricing results and offering an alternative to traditional Monte Carlo and lattice-based methods.
The literature on the pricing of barrier and lookback options is extensive and multifaceted: the development of Monte Carlo methods, hybrid models, and lattice-based approaches has significantly advanced the field, providing both computational efficiency and accuracy in the pricing of these complex options. As financial markets continue to evolve, further research into more accurate models incorporating jumps, volatility clustering, and fractional Brownian motion will be crucial for improving the pricing of exotic options. It is also worth to note that the option theory based on barrier monitoring can also be extended in the credit risk context, as shown in Agosto and Moretto, 2012.
This study can be conceptually divided into three parts: the first one explains in detail the continuous monitoring problem in a Monte Carlo engine based on the Black-Scholes-Merton pricing framework and how this can be solved through the Babsiri and Noel approach. The second part of the paper validates the methodology with first (standard lookback and barrier option) and second (soft barrier and double barrier option) generation exotic path dependent options. The bias introduced by standard Monte Carlo will be quantified and we will show that it can be zeroed through the implementation of Conditional Monte Carlo. The last part of the study is devoted to a concrete market case: an investment certificate of the Bonus Cap type (ACEPI certificates map, 2024) characterized by continuous barrier monitoring will be evaluated.

2) Methodology

The Brownian Bridge, also known in the literature as Tied Down Brownian Motion, is described for deriving a Monte Carlo method suitable for valuing path-dependent options, i.e. the Conditional Monte Carlo. The key idea of this approach, which avoids simulation bias distortion, is to directly derive the extreme value (S_{min} o S_{max}) from a probability distribution valid for the type of simulation performed. This methodology, based on the application of the Brownian Bridge and the reflection principle of Brownian motion, eliminates the need for a very fine partition for discretization. It proves to be effective both in terms of the accuracy of the fair value of the derivative, and for computational time performance. To keep the discussion manageable, only the fundamental steps needed for this characterization are presented, with the formal proof omitted but referenced in the bibliography (Kloeden and Platen, 1992). The next step is to determine the probability distribution governing the simulation of the maximum value potentially achieved by the underlying asset.

$$dZ_t = d\ln[S(t)] = \left(r - \frac{\sigma^2}{2}\right)dt + \sigma dW_t \rightarrow S_t = S_0 \exp\left[\left(r - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t\right] \quad (1)$$

Where $S(t)$ is the price of the underlying asset at time t, S_0 is the initial price of the underlying asset at time $t = 0$, r is the risk-free interest rate, σ is the volatility of the underlying asset, dW_t is the increment of a Wiener process (standard Brownian motion), dZ_t is the change in the logarithm of the asset price, representing the stochastic component of the process, and dt is the increment of time.
Considering equation (1) the probability that a path of Z starts at Z_i at time t_i and ends at Z_{i+1} at time t_{i+1} is given by the probability density function of the transition in (2) where $a = r - \frac{\sigma^2}{2}$ is the drift of the stochastic process.

$$p\{Z(t_i) = Z_i, Z(t_{i+1}) = Z_{i+1}\} = \frac{1}{\sigma\sqrt{2\pi\Delta t}} \exp\left[-\frac{(Z_{i+1} - Z_i - a\Delta t)^2}{2\sigma^2\Delta t}\right] \quad (2)$$

Given the initial and final values of Z, the probability that this path crosses a certain level b, which is the barrier $b = \ln(B)$ in the case of Barrier Options or is equivalent to the extreme variable value of the asset $b = \ln(S_{max})$ in the case of Lookback Options, within a time interval $t_i < \tau_b < t_{i+1}$ is as shown in (3).

$$p\{t_i < \tau_b < t_{i+1} | Z_i, Z_{i+1}\} = \frac{p\{t_i < \tau_b < t_{i+1}, Z(t_i) = Z_i, Z(t_{i+1}) = Z_{i+1}\}}{p\{Z(t_i) = Z_i, Z(t_{i+1}) = Z_{i+1}\}} \quad (3)$$

The denominator of this fraction is given by the equation (2). The numerator is computed as in (4)

$p\{t_i < \tau_b < t_{i+1} | Z_i, Z_{i+1}\} =$

$$p\{t_i < \tau_b < t_{i+1}\} \cdot p\{Z(t_i) = Z_i, Z(t_{i+1}) = Z_{i+1} | t_i < \tau_b < t_{i+1}\} =$$

$$p\{t_i < \tau_b < t_{i+1}\} \cdot p\{Z(t_i) = Z_i, Z(t_{i+1}) = Z_{i+1}^R | t_i < \tau_b < t_{i+1}\} =$$

$$= p\{Z(t_i) = Z_i, Z(t_{i+1} = Z_{i+1}^R, t_i < \tau_b < t_{i+1}\} \quad (4)$$

In equation (4), the reflection principle of Brownian motion is applied. According to this principle, the probability that a path Z starts at (t_i, Z_i) and ends at (t_{i+1}, Z_{i+1}) while crossing the level b is the same as that of starting from the same initial point and ending at (t_{i+1}, Z_{i+1}^R), where Z^R is the level of reflection of Z along b.

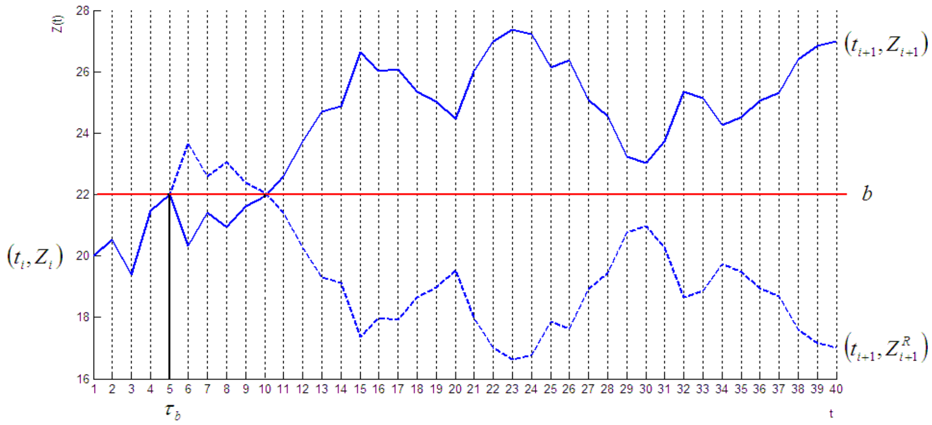


Figure 1: The reflection principle of a Brownian motion

$$p\{Z(t_i) = Z_i, Z(t_{i+1} = Z_{i+1}^R, t_i < \tau_b < t_{i+1}\} \quad (5)$$

The probability shown in (5) can be expressed by the following probability density function in (6) where $N(x)$ is the normal density distribution.

$$p\{t_i < \tau_b < t_{i+1}, Z_i, Z_{i+1}\} = \exp\left[\frac{2a(b-Z_i)}{\sigma^2}\right]\frac{1}{\sigma\sqrt{\Delta t}} N\left(-\frac{2b-Z_{i+1}-Z_i+a\Delta t}{\sigma\sqrt{\Delta t}}\right) \quad (6)$$

$$p\{Z(t_i) = Z_i, Z(t_{i+1}) = Z_{i+1}\} = \frac{1}{\sigma\sqrt{2\pi\Delta t}} \exp\left[-\frac{(Z_{i+1}-Z_i-a\Delta t)^2}{2\sigma^2\Delta t}\right] \quad (7)$$

By substituting equations and inserting (5) into (7), we obtain the desired conditional probability.

$$p\{t_i < \tau_b < t_{i+1} | Z_i, Z_{i+1}\} = \frac{\exp\left[\frac{2a(b-Z_i)}{\sigma^2}\right]\frac{1}{\sigma\sqrt{2\pi\Delta t}}\exp\left[-\frac{(2b-Z_{i+1}-Z_i+a\Delta t)^2}{2\sigma^2\Delta t}\right]}{\frac{1}{\sigma\sqrt{2\pi\Delta t}}\exp\left[-\frac{(Z_{i+1}-Z_i-a\Delta t)^2}{2\sigma^2\Delta t}\right]} \quad (8)$$

Performing the appropriate simplifications and calculations, we obtain equation (9).

$$p\{t_i < \tau_b < t_{i+1} | Z_i, Z_{i+1}\} = \exp\left[-\frac{2(b-Z_{i+1})(b-Z_i)}{\sigma^2\Delta t}\right] \quad (9)$$

$$p\{t_i < \tau_b < t_{i+1} | S_i, S_{i+1}\} = \exp\left(-\frac{2[\ln(B)-\ln(S_i)][\ln(B)-\ln(S_{i+1})]}{\sigma^2\Delta t}\right) = \exp\left(\frac{1}{\sigma^2\Delta t}\left[2\ln\left(\frac{B}{S_i}\right)\ln\left(\frac{S_{i+1}}{B}\right)\right]\right) \quad (10)$$

Rewriting the equation in terms of the underlying asset S, substituting $Z_i = \ln(S_i)$, $Z_{i+1} = \ln(S_{i+1})$, and $b = \ln(B)$, we obtain equation (10).
Therefore, to simulate the maximum value achieved by the asset in the interval $[t_i, t_{i+1}]$ given the Brownian Bridge endpoints S_i and S_{i+1}, it is sufficient to generate a uniformly distributed random variable $u \in U[0,1]$ and set it equal to the expression in (8).

$$u = \exp\left(\frac{1}{\sigma^2\Delta t}\left[2\ln\left(\frac{S_{max}}{S_i}\right)\ln\left(\frac{S_{i+1}}{S_{max}}\right)\right]\right) \quad (11)$$

Analytically solving equation (11) yields a direct expression for simulating the price of S_{max} as shown in (12).

$$S_{max} = \exp\left[\frac{\ln(S_{i+1}\cdot S_i)+\sqrt{\left[\ln\left(\frac{S_{i+1}}{S_i}\right)\right]^2-2\left(\frac{\sigma^2}{S_i}\right)^2\Delta t\ln(u)}}{2}\right] \quad (12)$$

To streamline this into a computationally more efficient distribution for programming environments, Babsiri and Noel (1998) suggested focusing on the log ratio between the simulated right endpoint of the Brownian Bridge $S_{i+1} = S(T)$ and the known left endpoint $S_i = S(0)$, defining this quantity as x in (13).

$$x = \ln\left(\frac{S(T)}{S(0)}\right) \quad (13)$$

Given the known distribution of this log ratio, $N\left(r\Delta t, \sigma\sqrt{\Delta t}\right)$, it is possible to estimate the conditional probability of the maximum value x using similar logical steps as before (see Equation (14)).

$$p\left\{\max \ln\left(\frac{S(t)}{S(0)}\right) \leq y : \ln\left(\frac{S(T)}{S(0)}\right) = x, t \in [0,T]\right\} = 1 - \exp\left[\frac{2y(x-y)}{\sigma^2 T}\right] \quad (14)$$

Thus, to simulate the maximum value x in the interval $t \in [0,T]$ it is sufficient to generate a uniformly distributed random variable $u \in U[0,1]$and set it equal to the one in (15).

$$u = 1 - \exp\left[\frac{2y(x-y)}{\sigma^2 T}\right] \to y_{MAX} = \frac{x + \sqrt{x^2 - 2\sigma^2 T \ln(1-u)}}{2} \quad (15)$$

The introduced transformation provides a distribution expression for the maximum value (15) that is more efficient to process compared to equation (12), making it preferable, especially when many simulations are needed.
To determine the probability distribution of the minimum value of x, we simply compute the complementary probability of (11) and set the probability to $u \in U[0,1]$.

$$p\left\{\min \ln\left(\frac{S(t)}{S(0)}\right) \leq y : \ln\left(\frac{S(T)}{S(0)}\right) = x\right\} = 1 - p\left\{\min \ln\left(\frac{S(t)}{S(0)}\right) \leq y : \ln\left(\frac{S(T)}{S(0)}\right) = x\right\} = \exp\left[\frac{2y(x-y)}{\sigma^2 T}\right] \quad (16)$$

$$u = \exp\left[\frac{2y(x-y)}{\sigma^2 T}\right] \to y_{MIN} = \frac{\left(x - \sqrt{x^2 - 2\sigma^2 T \ln(u)}\right)}{2} \quad (17)$$

By inverting this expression, we obtain (17), the simulation for the desired minimum value.

3) Empirical Results and Discussion

In this section, we present our analysis, applying both traditional and Conditional Monte Carlo methods to four types of exotic, path-dependent options: standard barrier, lookback, soft barrier, and double barrier options (Haug, 2007). For each option type, we evaluate and compare the pricing performance of both approaches, focusing on key metrics such as accuracy, convergence rate, and computational efficiency. It is important to highlight that the proposed methodology is valid when the dynamics which rules the underlying process is a Geometric Brownian Motion. In fact, this also constitutes a fundamental hypotheses of the Black-Scholes-Merton pricing framewok. We also discuss the implications of these findings in terms of their practical applicability, robustness in different market scenarios, and relevance to risk management strategies. Each subsection (3.1, 3.2, 3.3, and 3.4) provides a detailed assessment of its unique characteristics and the results obtained through the Monte Carlo simulations.

3.1) Standard Barrier Options

Barrier options are the first class of exotic options we studied. They are of course part of the path dependent options, and their payoff depends on whether the underlying asset price reaches a predetermined barrier level during the life of the option. This feature makes barrier options more complex than vanilla options, as the path of the underlying asset price - not just its final price - determines the option payoff. The barrier can either activate the option (knock-in) or terminate it (knock-out).

Barrier options are widely traded in the over-the-counter (OTC) market and they are popular because they often have lower premiums compared to standard options. This lower cost reflects the reduced likelihood of the option paying out, given the presence of the barrier. The level of the barrier has a significant impact on the option value, making barrier options highly sensitive to the volatility of the underlying asset.

Barrier options can be broadly categorized into two main types:

Knock-Out Options: These options cease to exist if the underlying asset price breaches the barrier level, H.
Knock-out options can further be classified into:

- Down-and-Out Call/Put: The option is knocked out (terminated) if the asset price falls to or below a certain barrier level, H, with $H < S_0$.
- Up-and-Out Call/Put: The option is knocked out if the asset price rises to or above a certain barrier level.

Knock-In Options: These options only come into existence if the underlying asset price breaches the barrier level. Knock-in options are categorized as:

- Down-and-In Call/Put: The option becomes active if the asset price falls to or below a certain barrier level, $H < S_0$.
- Up-and-In Call/Put: The option becomes active if the asset price rises to or above a certain barrier level.

Valuing barrier options involves more complexity than valuing standard options due to the path-dependency of the payoff. The valuation must account for the probability that the barrier will be breached. The Black-Scholes model, often used for vanilla options, can be adapted for barrier options with the addition of specific adjustments to account for the barrier feature.
If $H \leq K$, the current value c_{DI} for a Down-and-in call is:

$$c_{DI} = S_0 \cdot \exp(-qt) \cdot \left(\frac{H}{S_0}\right)^{2\lambda} N(y) - K \exp(-rt) \cdot \left(\frac{H}{S_0}\right)^{2\lambda-2} N\left(y - \sigma\sqrt{T}\right) \quad (18)$$

$$\lambda = \frac{r-q+\frac{\sigma^2}{2}}{\sigma^2} \quad (19), \quad y = \frac{\ln\left(\frac{H^2}{S_0 K}\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{(T)} \quad (20)$$

An ordinary call (c) equals the sum of the corresponding down-and-in and down-and-out calls. A down-and-out call option is a standard call option that ceases to exist if the underlying asset price falls to a barrier level H (where $H < S_0$, and S_0 is the initial price of the underlying asset). The value of this option, c_{DO}, can be derived as: $c_{DO} = c - c_{DI}$

Where:
c is the value of a standard European call option.

c_{DI} is the value of the corresponding down-and-in call option.

If the barrier is never breached, the down-and-out call option has a value at maturity. If the barrier is breached, the option ceases to exist, and the payoff is zero.
If $H > K$, the value of a down-and-out call is as shown in (23).

$$c_{DO} = S_0 \cdot \exp(-qt) \cdot N(x_1) - K \exp(-rt) \cdot N\left(x_1 - \sigma\sqrt{T}\right) - S_0 \cdot \exp(-qt) \left(\frac{H}{S_0}\right)^{2\lambda} N(y_1) + K \cdot \exp(-rt) \left(\frac{H}{S_0}\right)^{2\lambda-2} N\left(y_1 - \sigma\sqrt{T}\right) \quad (21)$$

$$x_1 = \frac{\ln\left(\frac{S_0}{H}\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \quad (22), \quad y_1 = \frac{\ln\left(\frac{H}{S_0}\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \quad (23)$$

Similarly to the previous case, the value of a down-and-in call is given by $c_{DI} = c - c_{DO}$
Up-and-out calls are also knock-out options. They are ordinary calls that cease to exist when the price of the underlying asset rises to H, with $H > S_0$.
If $H \leq K$, the current value of an up-and-out call, c_{UO} is null and the current value of an up-and-in call, c_{UI}, is equal to vanilla call c.
In case $H > K$, the value for an up-and-in call is shown in equation (24).

$$c_{UI} = S_0 \cdot \exp(-qt) \cdot N(x_1) - K \exp(-rt) \cdot N(x_1 - \sigma\sqrt{T}) - S_0 \cdot \exp(-qt) \left(\frac{H}{S_0}\right)^{2\lambda} [N(-y) - N(-y_1)] + K \cdot \exp(-rt) \left(\frac{H}{S_0}\right)^{2\lambda-2} \left[N\left(-y + \sigma\sqrt{T}\right) - N\left(-y_1 + \sigma\sqrt{T}\right)\right] \quad (24)$$

while the current value of an up-and-out call is $c_{UO} = c - c_{UI}$.
Barrier puts work similarly to barrier calls but the direction of the price movement is reversed. Standard up-and-out puts cease to exist when the price of the underlying asset rises to H, with $H > S_0$. Standard up-and-in puts only start to exist when the price of the underlying asset rises to H, with $H > S_0$.
If $H > K$, the current value of an up-and-in put, p_{UI} is shown in (25).

$$p_{UI} = -S_0 \cdot \exp(-qT) \left(\frac{H}{S_0}\right)^{2\lambda} N(-y) + K \cdot \exp(-rT) \left(\frac{H}{S_0}\right)^{2\lambda-2} N\left(-y + \sigma\sqrt{T}\right) \quad (25)$$

While the current value of an up-and-out put is $p_{UO} = p - p_{UI}$.
In case $H \leq K$, the current value of an up-and-out put is equal to (26).

$$p_{UO} = -S_0 \cdot \exp(-qT)\, N(-x_1) + K \cdot \exp(-rT)\, N\left(-x_1 + \sigma\sqrt{T}\right) + S_0 \cdot \exp(-qT) \left(\frac{H}{S_0}\right)^{2\lambda} N(-y_1) - K \cdot \exp(-rT)\, N\left(-y_1 + \sigma\sqrt{T}\right) \left(\frac{H}{S_0}\right)^{2\lambda-2} \quad (26)$$

while the current value of an up-and-in put is $p_{UI} = p - p_{UO}$.
Down-and-out puts cease to exist when the price of the underlying asset falls to H, with $H < S_0$. Down-and-in puts only start to exist when the price of the underlying asset falls to H, with $H < S_0$.
If $H \geq K$, the current value, p_{DO}, of a down-and-out put is zero and the current value, p_{DI}, of a down-and-in put is equal to p. If $H < K$, the current value of a down-and-in put, p_{DI} is as shown in (27) and the current value of a down-and-out put is $p_{DO} = p - p_{DI}$.

$$p_{DI} = -S_0 \cdot \exp(-qT)\, N(-x_1) + K \cdot \exp(-rT)\, N\left(-x_1 + \sigma\sqrt{T}\right) + S_0 \cdot \exp(-qT) \left(\frac{H}{S_0}\right)^{2\lambda} [N(y) - N(y_1)] - K \cdot \exp(-rT) \left(\frac{H}{S_0}\right)^{2\lambda-2} \left[N\left(y - \sigma\sqrt{T}\right) - N\left(y_1 - \sigma\sqrt{T}\right)\right] \quad (27)$$

All the valuation formulas for the barrier options presented are based on the assumption that the probabilistic distribution of the share price in a future instant of time is log-normal (Di Franco, Polimeni and Proietti, 2002).
A crucial aspect of barrier options is how frequently the underlying asset price is observed. The formulas presented earlier assume continuous observation, which is an idealization. In reality, the price is often observed discretely - daily, weekly, or at other intervals. This introduces the need for adjustments in valuation to account for the possibility of the barrier being breached between observation points.

Broadie, Glasserman and Kou (1999) have developed an approximation of the formula to take into account the discretization of the observation frequency. The correction factor proposed by these researchers is based on the modification to be made, for each observation, on the level of the barrier with: $H_U = H \cdot \exp\left(\beta\sigma\sqrt{\Delta t}\right)$ if the barrier is an upper-bound for the asset underlying the option. If the barrier represents a lower-bound, the adjustment is $H_D = H \cdot \exp\left(-\beta\sigma\sqrt{\Delta t}\right)$. Δt is the time that elapses between the instants of observation of the barrier. $\beta = \frac{\zeta(0.5)}{\sqrt{2\pi}} \approx 0.5826$, where $\zeta(\cdot)$ is the Riemann zeta-function.
In order to implement this in a programming environment, it is useful to rearrange the previous formulas of Reiner and Rubinstein (1991) according to the classification proposed by Rich (1994).
This pricing procedure provides for the use of the cost-of-carry, $b = r - q$, and the rebate feature, (R), where the option holder receives a fixed amount if the barrier is breached, and the option is knocked out. This rebate can be structured as either a cash payment or a payment in the form of an asset. The inclusion of a rebate alters the valuation, as it provides a fallback payoff, reducing the risk for the option holder.

$$A = \phi S \cdot \exp[(b-r)T]\, N(\phi x_1) - \phi K \cdot \exp(-rT)\, N\left(\phi x_1 - \phi\sigma\sqrt{T}\right) \quad (28)$$

$$B = \phi S \cdot \exp[(b-r)T]\, N(\phi x_2) - \phi K \cdot \exp(-rT)\, N\left(\phi x_2 - \phi\sigma\sqrt{T}\right) \quad (29)$$

$$C = \phi S \cdot \exp[(b-r)T] \left(\frac{H}{S}\right)^{2(\mu+1)} N(\eta y_1) - \phi K \cdot \exp(-rT) \left(\frac{H}{S}\right)^{2\mu} N\left(\eta y_1 - \eta\sigma\sqrt{T}\right) \quad (30)$$

$$D = \phi S \cdot \exp[(b-r)T] \left(\frac{H}{S}\right)^{2(\mu+1)} N(\eta y_2) - \phi K \cdot \exp(-rT) \left(\frac{H}{S}\right)^{2\mu} N\left(\eta y_2 - \eta\sigma\sqrt{T}\right) \quad (31)$$

$$E = R \cdot \exp(-rT) \left[N\left(\eta x_2 - \eta\sigma\sqrt{T}\right) - \left(\frac{H}{S}\right)^{2\mu} N\left(\eta y_2 - \eta\sigma\sqrt{T}\right)\right] \quad (32)$$

$$F = R \cdot \left[\left(\frac{H}{S}\right)^{\mu+\lambda} N(\eta z) + \left(\frac{H}{S}\right)^{\mu-\lambda} N\left(\eta z - 2\eta\lambda\sigma\sqrt{T}\right)\right] \quad (33)$$

Where:

$$x_1 = \frac{\ln\left(\frac{S}{K}\right)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \ (34), \qquad x_2 = \frac{\ln\left(\frac{S}{H}\right)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \ (35), \qquad y_1 = \frac{\ln\left(\frac{H^2}{SK}\right)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \ (36)$$

$$y_2 = \frac{\ln\left(\frac{H}{S}\right)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \ (37), \qquad z = \frac{\ln\left(\frac{H}{S}\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \ (38),$$

$$\mu = \frac{b - \frac{\sigma^2}{2}}{\sigma^2} \ (39), \quad \lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}} \ (40)$$

Down-and-in call $S > H$
Pay-off: max $(S - K; 0)$ if $S \leq H$ before T otherwise R at maturity.

$$c^{DI}_{K>H} = C + E \ (41), \ \eta = +1, \ \phi = +1 \ (42)$$

$$c^{DI}_{K<H} = A - B + D + E \ (43), \ \eta = +1, \ \phi = +1 \ (44)$$

Up-and-in call $S < H$
Pay-off: max $(S - K; 0)$ if $S \geq H$ before T otherwise R at maturity.

$$c^{UI}_{K>H} = A + E \ (45), \ \eta = -1, \phi = +1 \ (46)$$

$$c^{UI}_{K<H} = B - C + D + E \ (47) \qquad \eta = -1, \phi = +1 \ (48)$$

Down-and-in put $S > H$
Pay-off: $\max(K - S; 0)$ if $S \leq H$ before T otherwise R at maturity.

$$p^{DI}_{K>H} = B - C + D + E \ (49) \qquad \eta = +1, \phi = -1 \ (50)$$
$$p^{DI}_{K<H} = A + E \ (51) \qquad \eta = +1, \phi = -1 \ (52)$$

Up-and-in put $S < H$
Pay-off: $\max(K - S; 0)$ if $S \geq H$ before T otherwise R at maturity.

$$p^{UI}_{K>H} = A - B + D + E \ (53) \qquad \eta = -1, \phi = -1 \ (54)$$
$$p^{UI}_{K<H} = C + E \ (55) \qquad \eta = -1, \phi = -1 \ (56)$$

Down-and-out call S > H
Pay-off: $\max(S-K;0)$ if S > H before T otherwise R at the hit.

$$c^{DO}_{K>H} = A - C + F \quad (57) \qquad \eta = +1, \phi = +1 \quad (58)$$
$$c^{DO}_{K<H} = B - D + F \quad (59) \qquad \eta = +1, \phi = +1 \quad (60)$$

Up-and-out call S < H
Pay-off: $\max(S-K;0)$ if S < H before T otherwise R at the hit.

$$c^{UO}_{K>H} = F \quad (61) \qquad \eta = -1, \phi = +1 \quad (62)$$
$$c^{UO}_{K<H} = A - B + C - D + F \quad (63) \qquad \eta = -1, \phi = +1 \quad (64)$$

Down-and-out put $S > H$
Pay-off: $\max(K-S;0)$ if $S > H$ before T otherwise R at the hit.

$$p^{DO}_{K>H} = A - B + C - D + F \quad (65) \qquad \eta = +1, \phi = -1 \quad (66)$$
$$p^{DO}_{K<H} = F \quad (67) \qquad \eta = +1, \phi = -1 \quad (68)$$

Up-and-out put S < H
Pay-off: $\max(K-\mathrm{S};0)$ if S < H before T otherwise R at the hit.

$$p^{UO}_{K>H} = B - D + F \quad (69) \qquad \eta = -1, \phi = -1 \quad (70)$$
$$p^{UO}_{K<H} = A - C + F \quad (71) \qquad \eta = -1, \phi = -1 \quad (72)$$

3.1.1) Crude Monte Carlo application

Barrier options are sensitive to the path taken by the underlying asset, especially in relation to the barrier level. The assumption of continuous monitoring - where the asset price is constantly observed - simplifies the theoretical valuation of these options but it is impractical in real-world applications. Instead, the asset price is typically observed at discrete intervals, such as daily or weekly. This discrete monitoring can lead to different outcomes compared to continuous monitoring, thus influencing the estimated option price.
To implement a Crude Monte Carlo simulation for pricing a barrier option, we need to follow these steps:

1. **Model the Asset Price Path**: The asset price is typically modeled using a Geometric Brownian Motion (GBM), which follows the stochastic differential equation (73).
$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (73)$$
2. **Simulate Asset Paths**: We need to generate multiple simulations of the asset price path over the life of the option, taking into account the discrete monitoring points. The time steps Δt between monitoring points are crucial. For example, if we observe the price daily over a year, we have 252 steps (assuming 252 trading days).
3. **Check the Barrier Condition**: For each simulated path, we need to check if the barrier level is breached at any monitoring point. Depending on the type of barrier option (knock-in or knock-out), this will determine whether the option is activated or deactivated.
4. **Calculate the Payoff for Each Path**: After checking the barrier, we calculate the payoff for each simulated path. For a knock-out option, the payoff is zero if the barrier is breached. For a knock-in option, the payoff is calculated only if the barrier is breached.
5. **Average Payoffs Across Simulations**: The option price is then estimated as the discounted average of the payoffs from all simulations.

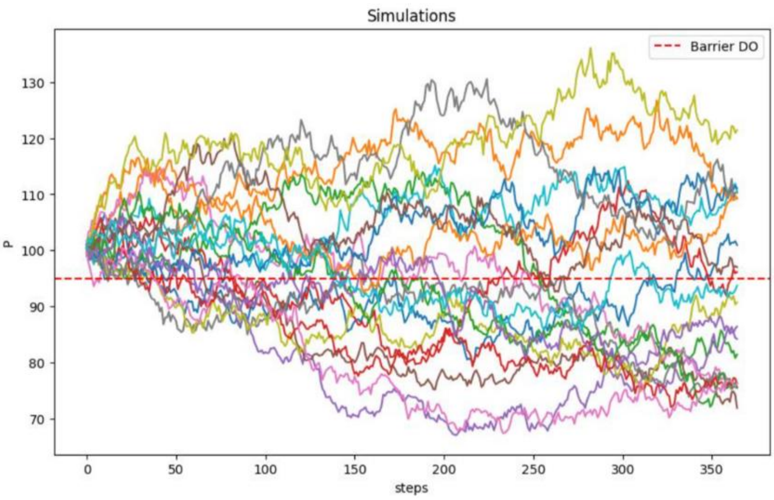


Figure 2: Different paths simulated through the Monte Carlo model

The key aspect to explore using Monte Carlo simulation is how different monitoring frequencies affect the barrier option price. A more frequent monitoring (e.g., daily) better approximates the continuous monitoring assumption, while a less frequent monitoring (e.g., weekly or monthly) may lead to different outcomes.
To explore this bias, a Crude Monte Carlo simulation has been implemented, to price a down-and-out call option under varying monitoring frequencies: 24 hours (daily), 1 hour, 30 minutes, and 15 minutes. The purpose of these experiments was to investigate how the choice of a monitoring interval impacts the estimated option price and to highlight the extent of the bias introduced by less frequent observations.
For the simulation, a Geometric Brownian Motion model has been used to generate the price paths of the underlying asset. The key parameters - initial stock price (S_0), strike price (K), barrier level (H), volatility (σ), risk-free rate (r), and time to maturity (T) - were kept constant across all trials to ensure consistency in the results. The only variable that was adjusted was the frequency at which the asset price was monitored to determine whether it breached the barrier.
The monitoring frequencies have been set at twenty-four hours, one hour, thirty minutes and fifteen minutes. The number of simulations at each iteration are set to 10.000; the loop went for 200 iterations. Each scenario was run through the Monte Carlo simulation to estimate the down-and-out barrier option price. The results were then aggregated and compared to understand the impact of different monitoring intervals on the option estimated value. The settings used to conduct the study are as follows:

$$S_0 = 100, K = 100, T = 1.0, r = 0.05, q = 0.02, \sigma = 0.2, H = 95$$

The exact price of this Down-and-Out call option is 4.8835.
The simulation results reveal a clear trend: as the monitoring frequency increases, the estimated price of the down-and-out option decreases, converging toward the theoretical value expected under continuous monitoring. This is due to the increased likelihood of the barrier being breached when the asset price is observed more frequently.

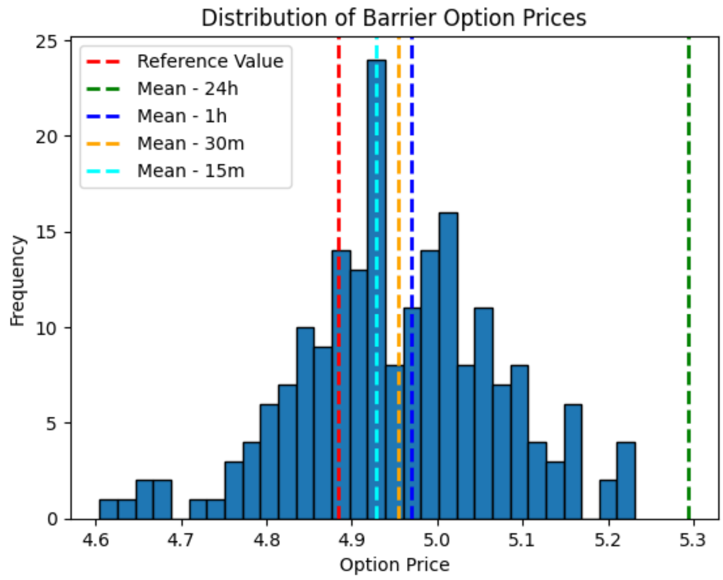


Figure 3: Different monitoring frequencies of Monte Carlo for Barrier Option Prices

Daily Monitoring (24 hours): This scenario produced the highest estimated option price. With only 24 observations across the life of the option, there were fewer opportunities for the price to hit the barrier, resulting in a lower probability of the option being knocked out and, therefore, a higher price.
Hourly Monitoring (1 hour): the option price was lower than the one computed with daily monitoring, reflecting the higher chance of the barrier being breached.
30-Minute Monitoring: As expected, the price continued to decrease with a more frequent monitoring, showing greater alignment with the continuous monitoring assumption.
15-Minute Monitoring: This scenario yielded the lowest estimated option price, most closely approximating the theoretical value, as the frequent checks made it more likely for the asset price to breach the barrier.

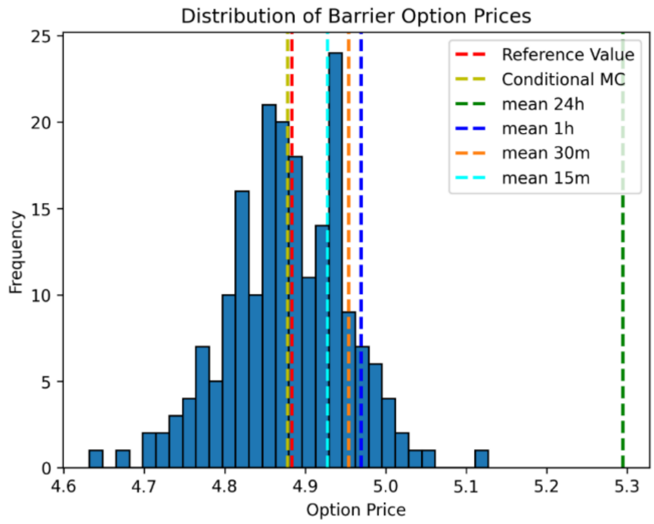
The results underscore the importance of accounting for a discretization bias in barrier option pricing. Traders and risk managers relying on less frequent monitoring may overestimate the value of a down-and-out barrier option, leading to potential mispricing and exposure to unanticipated risks. Conversely, more frequent monitoring, while computationally intensive, provides a more accurate estimate that better reflects the true risk profile of the option.
This bias is particularly relevant in markets where high-frequency trading and rapid price fluctuations are common. In such environments, the likelihood of the barrier being breached increases, making it crucial to adopt a monitoring strategy that closely approximates continuous observation.
The Crude Monte Carlo simulation results demonstrate the significant impact of monitoring frequency on the estimated price of down-and-out barrier options. By systematically reducing the time interval between observations - from 24 hours to 15 minutes - it becomes evident that discretization bias can lead to overvaluation when the barrier is monitored less frequently. These findings highlight the necessity for market participants to carefully consider the frequency of monitoring when pricing and managing barrier options, especially in fast-moving markets.

3.1.2) Conditional Monte Carlo application

In the context of barrier options, the Conditional Monte Carlo method is particularly suitable. It leverages the conditional expectation of the payoff given that the barrier has not been breached up to the current time. This approach reduces the noise in the simulation, as it only focuses on paths that are relevant to the option final payoff, thereby accelerating the convergence to the true option price. To demonstrate this, a Conditional Monte Carlo method has been implemented as well, on the same options analysed in the previous sections. The parameters used - initial stock price (S_0), strike price (K), barrier level (H), volatility (σ), risk-free rate (r), and time to maturity (T) - remained consistent with those used in the Crude Monte Carlo simulations. The goal is to highlight the improvements in both speed and precision when using the Conditional Monte Carlo method. The key difference between the two methods lies in how they simulate the price paths of the underlying asset:

Crude Monte Carlo: This method simulates numerous independent price paths of the underlying asset, checking whether the barrier has been breached at each time step. If the barrier is breached, the option becomes worthless for that path. This process, while straightforward, often requires many simulations to achieve a high degree of accuracy, as it does not account for any prior knowledge about the probability of the barrier being breached.
Conditional Monte Carlo: In contrast, the CMC method is conditional on the event that the barrier has not been breached by a certain time. This approach allows for the direct calculation of the expected payoff of the option, given that the price path is still valid (i.e., it has not hit the barrier). By focusing on these relevant paths, the CMC method reduces the variance of the estimated option price, leading to faster convergence and more accurate results with fewer simulations.

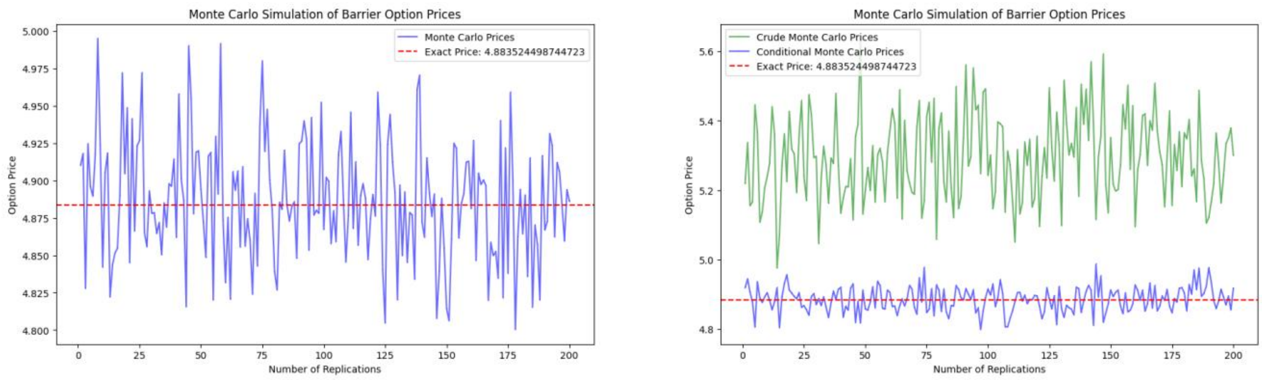
The Conditional method demonstrated a clear advantage in computational speed. The crude Monte Carlo method took increasingly longer with each reduction of the monitoring time window, reaching up to 870 minutes for the "15m" monitoring period. On the other hand, the Conditional Monte Carlo method only took one minute. By reducing the number of irrelevant paths (those where the barrier is breached early), the Conditional method required significantly fewer simulations to reach a given level of accuracy. This reduction in computational effort translates directly into faster runtimes, making the CMC method more suitable for real-time pricing and risk management applications where speed is critical.
In terms of precision, the Conditional method consistently produced more accurate estimates of the down-and-out barrier option price. The reduction in variance achieved by being conditional on the relevant paths meant that the option prices estimated by the Conditional method had a much narrower confidence interval compared to those produced by the Crude Monte Carlo method. This precision is particularly valuable when pricing options in volatile markets, where small errors can lead to significant financial consequences.



	Mean	Std. Dev
24h	5.2945	0.1230
1h	4.9690	0.1187
30m	4.9537	0.1180
15m	4.9277	0.1257
Cond.	4.8828	0.0413

Figure 4: Conditional Monte Carlo compared with the Crude Monte Carlo for Barrier Option Prices

A plot with the Conditional Monte Carlo results is shown in Figure 5. It has been run with two hundred replications loop, each with a hundred thousand simulations.



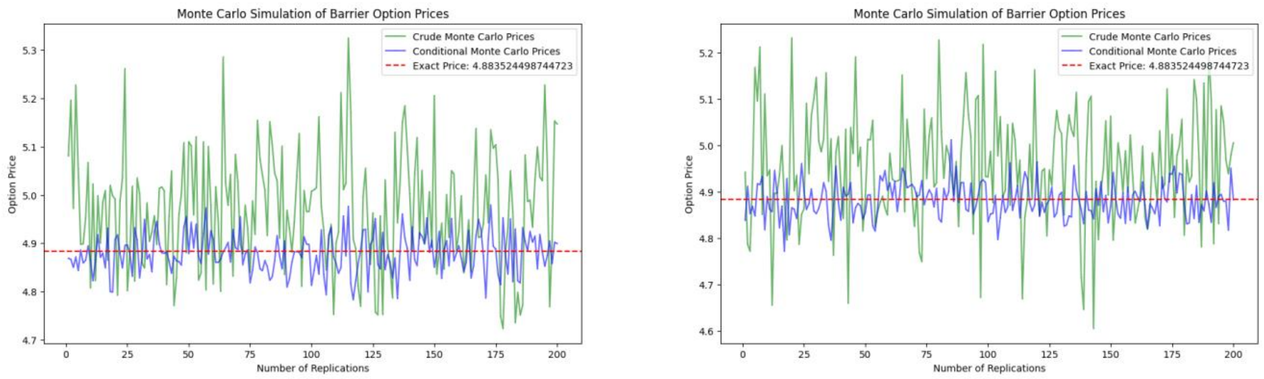


Figure 5A: 200 replications of Conditional Monte Carlo simulations for Barrier Option Prices
Figure 5B: 200 replications of Crude Monte Carlo simulations for Barrier Option Prices, 24 hours monitoring frequency
Figure 5C: 200 replications of Crude Monte Carlo simulations for Barrier Option Prices, 1 hour monitoring frequency
Figure 5D: 200 replications of Crude Monte Carlo simulations for Barrier Option Prices, 30 minutes monitoring frequency

3.2) Lookback Options

Lookback Options are sophisticated financial derivatives whose value depends on the minimum or maximum price reached by the underlying asset during the entire lifespan of the option. Unlike traditional options, where the strike price is fixed at the time of contract initiation, lookback options allow the holder to "look back" at the underlying asset price history to determine the optimal exercise price. There are two main types of lookback options: floating-strike lookback options and fixed-strike lookback options, each with its unique valuation method and payout structure.
In some cases, the observation period for the extreme values (maximum or minimum) of the underlying asset might be shorter than the full life of the option. These derivatives are known as Partial-Time Lookback Options, and they can be further categorized into partial-time fixed-strike and partial-time floating-strike lookback options. Given the complexity of these instruments, numerical methods are often required to accurately value them.
Lookback options are powerful tools for investors looking to hedge against or capitalize on significant price movements in the underlying asset. Their value derives from the most favorable price movements observed during the life of the option, making them particularly useful in volatile markets. However, the complexity of their valuation requires a deep understanding of the underlying models and assumptions, as well as a consideration of market conditions and the specific terms of the option contract.
In floating-strike lookback options, the strike price is not set in advance, but it is determined retrospectively, based on the minimum or maximum price reached by the underlying asset during the life of the option.
The final value of a floating-strike lookback call option is determined by the difference between the final price of the underlying asset S_T and the minimum price S_{min} recorded during the lifespan of the option. Mathematically, the payoff is expressed in (74).

$$c(S, S_{min}, T) = \max(S - S_{min}; 0) = S_T - S_{min} \quad (74)$$

Conversely, the final value of a floating-strike lookback put option depends on the difference between the maximum price S_{max} reached by the underlying asset during the life of the option and its final price S_T.

$$p(S, S_{max}, T) = \max(S_{max} - S; 0) = S_{max} - S_T \quad (75)$$

The payoff is given by equation (75). The valuation of these options can be complex and is often calculated using models like the Goldman-Sosin-Gatto (1979) and the Garman (1989) formulas. These models incorporate factors such as the cumulative normal distribution $N(\cdot)$ and the standard normal distribution $n(\cdot)$ to account for the stochastic behavior of asset prices.
The closed formula for the valuation of a Floating-Strike Lookback call option is shown in (76) and (77).

If $b \neq 0$

$$c = S \cdot \exp[(b-r)T]\, N(a_1) - S_{min} \cdot \exp(-rT)\, N(a_2) + \\ + S \cdot \exp(-rT) \frac{\sigma^2}{2b} \left[\left(\frac{S}{S_{min}} \right)^{-\frac{2b}{\sigma^2}} N\left(-a_1 + \frac{2b}{\sigma}\sqrt{T}\right) - \exp(bT)\, N(-a_1) \right] \quad (76)$$

If $b = 0$

$$c = S \cdot \exp(-rT)\, N(a_1) - S_{min} \cdot \exp(-rT)\, N(a_2) + \\ + S \cdot \exp(-rT)\, \sigma\sqrt{T}\{n(a_1) + a_1[N(a_1) - 1]\} \quad (77)$$

Where:

$$a_1 = \left(\frac{\ln\left(\frac{S}{S_{min}}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{(T)}} \right) \quad (78), \quad a_2 = a_1 - \sigma\sqrt{T} \quad (79)$$

Conversely, the exact formula for the put version of the option is in (80) and (81).

If $b \neq 0$

$$p = S_{max} \cdot \exp(-rT)\, N(-b_2) - S \cdot \exp[(b-r)T]\, N(-b_1) + S \cdot \exp(-rT)\, \frac{\sigma^2}{2b}\left[-\left(\frac{S}{S_{max}}\right)^{-\frac{2b}{\sigma^2}} N\left(b_1 - \frac{2b}{\sigma}\sqrt{T}\right) + \exp(bT)\, N(b_1)\right] \quad (80)$$

If $b = 0$

$$p = S_{max} \cdot \exp(-rT)\, N(-b_2) - S \cdot \exp[(b-r)T]\, N(-b_1) + S \cdot \exp(-rT)\, \sigma\sqrt{T}\{n(b_1) + N(b_1) b_1\} \quad (81)$$

$$b_1 = \frac{\ln\left(\frac{S}{S_{max}}\right) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (82), \quad b_2 = b_1 - \sigma\sqrt{T} \quad (83)$$

S: Current underlying price
r: Risk-free rate
q: Dividend yield
σ: volatility of underlying
T: Time to maturity
S_{min}: Minimum underlying price observed since the beginning of the contract
S_{max}: Maximum underlying price observed since the beginning of the contract

In contrast to floating-strike options, fixed-strike lookback options have a pre-determined strike price K that is set at the time the contract is entered into. The value of these options at expiration depends on the highest or lowest price reached by the underlying asset during the life of the option, relative to this fixed strike price.
The call Fixed-Strike Lookback option pays the maximum between the difference of the highest price observed during the life of the option S_{max} and the strike price K, or zero.

$$c(S, S_{max}, T) = \max(S_{max} - K; 0) \quad (84)$$

The payout for a fixed-strike lookback put is the maximum between the difference of the strike price K and the lowest price observed S_{min}, or zero.

$$p(S, S_{min}, T) = \max(K - S_{min}; 0) \quad (85)$$

Valuing fixed-strike lookback options often involves formulas developed by Conze and Viswanathan (1991), which account for the potential variance in outcomes depending on whether the strike price is greater than or less than the observed maximum or minimum prices.
For the call option:

If $K > S_{max}$

$$c = S \cdot \exp[(b-r)T]\, N(d_1) - K \cdot \exp(-rT) N(d_2) +$$
$$+ S \cdot \exp(-rT)\ \frac{\sigma^2}{2b}\left[-\left(\frac{S}{K}\right)^{-\frac{2b}{\sigma^2}} N\left(d_1 - \frac{2b}{\sigma}\sqrt{T}\right) - \exp(bT)\, N(d_1)\right] \quad (86)$$

Where:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (87), \quad d_2 = d_1 - \sigma\sqrt{T} \quad (88)$$

If $K \leq S_{max}$

$$c = \exp(-rT) \cdot (S_{max} - K) + S \cdot \exp[(b-r)T]\, N(e_1) - S_{max} \cdot \exp(-rT)\, N(e_2) +$$
$$+ S \cdot \exp(-rT)\ \frac{\sigma^2}{2b}\left[-\left(\frac{S}{S_{max}}\right)^{-\frac{2b}{\sigma^2}} N\left(e_1 - \frac{2b}{\sigma}\sqrt{T}\right) + \exp(bT)\, N(e_1)\right] \quad (89)$$

Where:

$$e_1 = \frac{\ln\left(\frac{S}{S_{max}}\right) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (90), \quad e_2 = e_1 - \sigma\sqrt{T} \quad (91)$$

The valuation for a Put option, on the other hand, is as follows:

If $K < S_{min}$

$$p = K \cdot \exp(-rT)\, N(-d_2) - S \cdot \exp[(b-r)T]\, N(-d_1) +$$
$$+ S \cdot \exp(-rT)\ \frac{\sigma^2}{2b}\left[\left(\frac{S}{K}\right)^{-\frac{2b}{\sigma^2}} N\left(-d_1 + \frac{2b}{\sigma}\sqrt{T}\right) - \exp(bT)\, N(-d_1)\right] \quad (92)$$

If $K \geq S_{min}$

$$p = \exp(-rT) \cdot (K - S_{min}) - S \cdot \exp[(b-r)T]\, N(-f_1) - S_{min} \cdot \exp(-rT)\, N(-f_2)$$
$$+S \cdot \exp(-rT)\ \frac{\sigma^2}{2b}\left[\left(\frac{S}{S_{min}}\right)^{-\frac{2b}{\sigma^2}} N\left(-f_1 + \frac{2b}{\sigma}\sqrt{T}\right) - \exp(bT)\, N(-f_1)\right] \quad (93)$$

Where $f_1 = \frac{\ln\left(\frac{S}{S_{min}}\right) + (b+\sigma^2/2)T}{\sigma\sqrt{T}}$ (94), $f_2 = f_1 - \sigma\sqrt{T}$ (95).

3.2.1) Crude Monte Carlo application

The Crude Monte Carlo method directly replicates the logic of a lookback option. In this approach, the daily prices of the underlying asset are simulated and stored in a vector. At the end of the simulation, the minimum or maximum value—depending on whether it is a call or a put option—is selected from the vector for use in calculating the payoff.

For the simulation, a Geometric Brownian Motion model has been used to generate the price paths of the underlying asset. The key parameters—initial stock price (S), strike price (K), volatility (σ), risk-free rate (r), and time to maturity (T)—were kept constant across all trials to ensure consistency in the results. The starting parameter of the function, M_0, is S_{max} in case of a Put option, S_{min} if the option valued is a Call. The only variable that was adjusted was the frequency at which the asset price was monitored.

The monitoring frequencies have been set at twenty-four hours, one hour, thirty minutes and fifteen minutes. As before, the number of simulations at each iteration are set to 10.000; the loop went for 200 iterations. Each scenario was run through the Monte Carlo simulation to estimate a Floating-Strike Lookback Put option price first, and then the method has been applied on the valuation of a Fixed-Strike Lookback Call. The results have been aggregated and compared to understand the impact of different monitoring intervals on the option estimated value. The settings used to conduct the study are as follows: $S = 120, M_0 = 130, T = 1.0, r = 0.4, q = 0.02, \sigma = 0.3$. The exact price of this option is 12.4819.

The simulation results reveal the same trend as the one shown in the application for the barrier options: as the monitoring frequency increases, the estimated price of the option increases, converging toward the theoretical value expected under continuous monitoring.

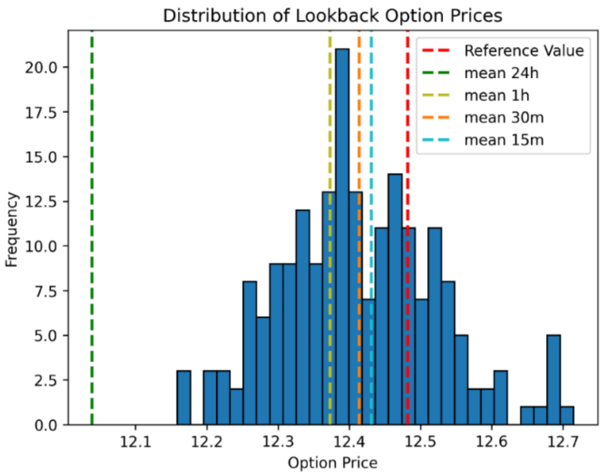


Figure 6: Crude Monte Carlo for Floating-Strike Lookback options at different monitoring frequencies

The settings applied to the function for the application of the Crude Monte Carlo for pricing a Fixed-Strike version of a Lookback Call option, are the following: $S = 100, M_0 = 100, T = 1.0, r = 0.4, q = 0.02, \sigma = 0.3$. The result of the closed formula valuation is 8.6626. Here, too, the same dynamic can be observed: as the monitoring frequency increases, the accuracy of the method becomes significantly better.

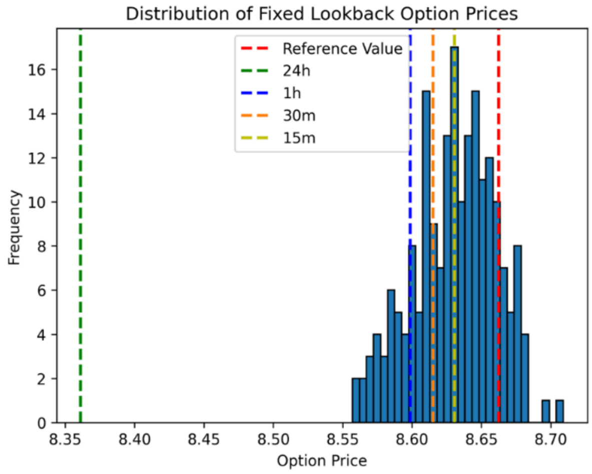
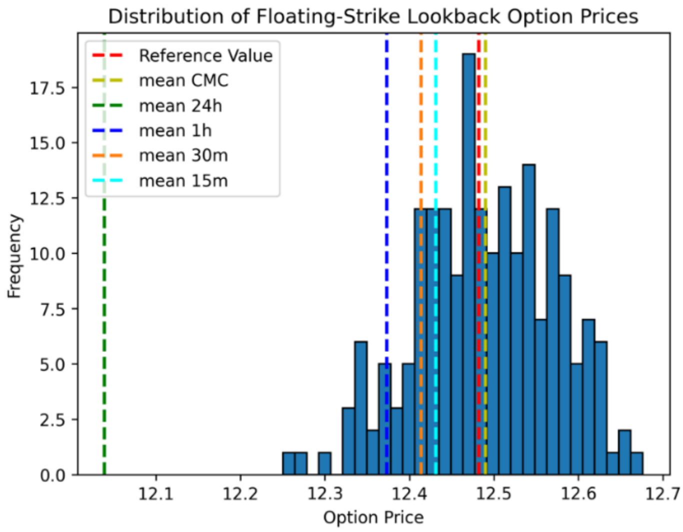


Figure 7: Crude Monte Carlo for Fixed-Strike Lookback options at different monitoring frequencies

The same kind of bias can be observed, as the distribution of the function set at a narrower monitoring frequency shows closer and closer results to the exact one. It can be shown that for lookback options, with the same number of iterations, the approximation is slightly less precise than the one obtained applying the method on the barrier options.

3.2.2) Conditional Monte Carlo application

Although this method accurately replicates the dynamics of the derivative, it is subject to numerical integration errors due to the inability to continuously monitor the underlying asset. Consequently, the Conditional Monte Carlo method, previously introduced in the section on standard barrier options, is often preferred. This method is convenient when the focus is solely on determining the extreme values that the underlying asset might reach within a given time frame, utilizing a numerical technique that adheres to the principles of the Brownian Bridge.
The Conditional method demonstrated the same advantages in both computational speed and precision. The crude Monte Carlo method took increasingly longer with each reduction of the monitoring time window, while the conditional Monte Carlo method only took one minute. By reducing the number of irrelevant paths, the Conditional method required far fewer simulations to reach a given level of accuracy. This reduction in computational effort translates directly into faster runtimes, making the CMC method the suitable choice here as well.



	Mean	Std. Dev
24h	12.0389	0.1226
1h	12.3731	0.1192
30m	12.4139	0.1105
15m	12.4312	0.1147
Cond.	12.4774	0.0413

Figure 8: Conditional Monte Carlo: comparison with the Crude Monte Carlo for Floating-Strike Lookback Option Prices

The graph with the Conditional Monte Carlo results is shown in Figure 9. This, too, has been set to run for two hundred replications, each with a set number of a hundred thousand simulations.

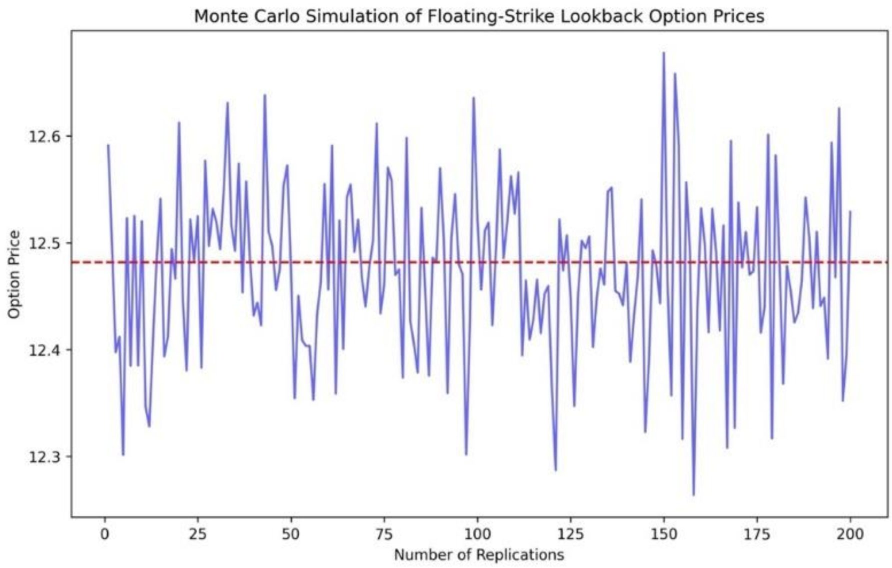
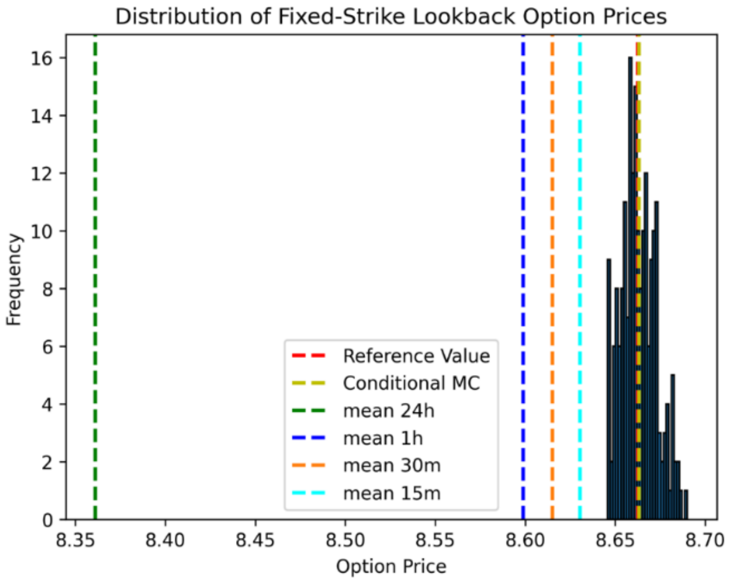


Figure 9: 200 replications of the Conditional Monte Carlo model for Floating-Strike Lookback Option prices

In terms of precision, the Conditional method consistently produced more accurate estimates for the lookback option. The option prices estimated by the Conditional method had a much narrower confidence interval compared to those produced by the Crude Monte Carlo method.



	Mean	Std. Dev
24h	8.3612	0.0387
1h	8.5987	0.0363
30m	8.6152	0.0424
15m	8.6304	0.0297
Cond.	8.6624	0.0209

Figure 10: Conditional Monte Carlo: comparison with the Crude Model for Fixed-Strike Lookback Option prices

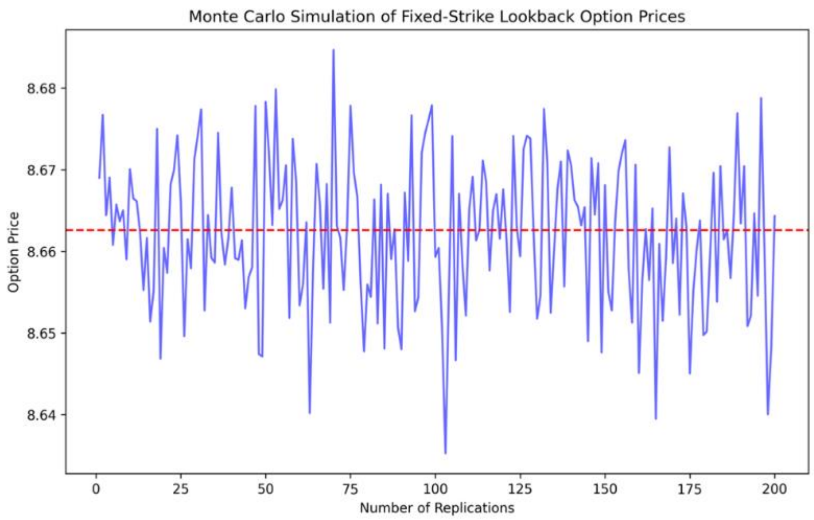


Figure 11: 200 replications of Conditional Monte Carlo on Fixed-Strike Lookback options

3.3) Soft Barrier Options

Another variant of the standard barrier option is the soft-barrier option. It is similar to the latter, but the barrier is no longer defined by a single level. Rather, it is a "soft range" between an upper level, U, and a lower level, L. The main difference between a soft and a standard barrier, is that soft-barrier options are knocked in – or out – proportionally. For instance, consider a soft down-and-out call with a current asset price of 100, with a soft barrier range from $U = 90$ to $L = 80$. If the lowest asset price during the lifetime is 86, then 40% of the call will be knocked out.
Hart and Ross (1994) introduced for the first time the closed formula that can be applied and used to find the fair value of the soft-down-and-in call and soft-up-and-in put options.

$$w = \frac{1}{U-L}\left\{\eta S e^{(b-r)T} S^{-2\mu} \frac{(SK)^{\mu+0.5}}{2(\mu+0.5)}\left[\left(\frac{U^2}{SK}\right)^{\mu+0.5} N(\eta d_1) - \lambda_1 N(\eta d_2) - \left(\frac{L^2}{SK}\right)^{\mu+0.5} N(\eta e_1) + \lambda_1 N(\eta e_2)\right] + \right.$$
$$\left. -\eta K e^{-rT} S^{-2(\mu-1)} \frac{(SK)^{\mu-0.5}}{2(\mu-0.5)}\left[\left(\frac{U^2}{SK}\right)^{\mu-0.5} N(\eta d_3) - \lambda_2 N(\eta d_4) - \left(\frac{L^2}{SK}\right)^{\mu-0.5} N(\eta e_3) + \lambda_2 N(\eta e_4)\right]\right\} \quad (96)$$

where η is set to 1 for a call and -1 for a put, and

$$d_1 = \frac{\ln\left(\frac{U^2}{SK}\right)}{\sigma\sqrt{T}} + \mu\sigma\sqrt{T} \ (97), \quad d_2 = d_1 - (\mu+0.5)\sigma\sqrt{T} \ (98)$$
$$d_3 = \frac{\ln\left(\frac{U^2}{SK}\right)}{\sigma\sqrt{T}} + (\mu-1)\sigma\sqrt{T} \ (99), \quad d_4 = d_3 - (\mu-0.5)\sigma\sqrt{T} \ (100)$$
$$e_1 = \frac{\ln\left(\frac{L^2}{SK}\right)}{\sigma\sqrt{T}} + \mu\sigma\sqrt{T} \ (101), \quad e_2 = e_1 - (\mu+0.5)\sigma\sqrt{T} \ (102)$$

$$e_3 = \frac{\ln\left(\frac{L^2}{SK}\right)}{\sigma\sqrt{T}} + (\mu - 1)\sigma\sqrt{T} \ (103), \ \ e_4 = e_3 - (\mu - 0.5)\sigma\sqrt{T} \ (104)$$
$$\lambda_1 = e^{-0.5\left[\sigma^2 T(\mu+0.5)(\mu-0.5)\right]} \ (105), \qquad \lambda_2 = e^{-0.5\left[\sigma^2 T(\mu-0.5)(\mu-1.5)\right]} \ (106)$$
$$\mu = \frac{b+\frac{\sigma^2}{2}}{\sigma^2} \ (107)$$

For the valuation of the price of a soft down-and-out call, the value of a soft down-and-in call must be subtracted to a standard call. Similarly, the value of a soft up-and-out put is equal to the value of a standard put, minus a soft up-and-in put.
Standard barrier options become increasingly difficult to delta hedge as the asset price nears the barrier.
This occurs due to an increase in gamma risk, which reflects how sensitive the option delta is to changes in the price of the underlying asset.
A higher gamma implies that even small fluctuations in the asset price can cause significant shifts in delta, making it harder to maintain an effective hedge.
Delta hedging refers to the strategy of adjusting the position in the underlying asset to neutralize the option price sensitivity (delta) to asset movements.
In contrast, soft-barrier options generally exhibit lower gamma risk, meaning their delta is less reactive to price changes, thus simplifying the hedging process.

3.3.1) Crude Monte Carlo application

A Monte Carlo method has been applied in the same way it has been applied on the other options seen previously.
The Crude Monte Carlo method directly replicates the working principle of a soft-barrier option. For soft-barrier options, the main feature is that the barrier is not hit instantly; instead, the payoff is determined by whether the asset price crosses a 'soft' boundary within a certain range or time window, rather than a rigid threshold. This is crucial for the simulation, as it allows the model to capture the probabilistic nature of the barrier behavior. The soft-barrier feature is embedded in the Monte Carlo framework by modifying the payoff function to account for the gradual approach to the barrier.
At each time step of the simulation, the algorithm checks whether the underlying asset price has crossed the soft-barrier range, storing this information to determine whether the option is activated or not. The final payoff is computed based on whether the asset price has crossed the barrier, and if so, how far it has moved within the soft-barrier zone. This process is repeated for many simulated paths, each representing a potential future evolution of the asset price.
Once all the paths are simulated, the average of the payoffs across all paths is calculated. This average represents the expected payoff of the option under the assumed stochastic process.
A Down-and-Out Call has been used for the simulation. The settings of the option price used to conduct the Monte Carlo simulations are the followings: $S = 100, K = 100, U = 95, L = 90, T = 0.5, r = 0.1, b = 0.05, \sigma = 0.2$.
The precision of the Crude Monte Carlo approach depends heavily on the number of simulations performed. A higher number of simulations typically leads to more accurate pricing, but it increases the computational cost. In practice, variance reduction techniques may be employed alongside the basic Monte Carlo algorithm to improve efficiency and reduce the error margin, while maintaining the accuracy of the price estimate (Bottasso *et al.*, 2023).
The monitoring frequencies have been set at twenty-four hours and one hour. Differently from before, only twenty-four and one-hour frequencies have been set for two reasons: the first is that approaching from twenty-four to one hour – the widest "jump" – shows only minimal improvements in the results of the simulations; the other is that calculation times of the higher frequencies would have been very long. As always, the number of simulations at each iteration is set to 10.000; the loop went for 200 iterations. Each scenario was run through the Monte Carlo simulation to estimate a Soft Down-and-Out Call option price. The paths have been shown in a different kind of plot to show the bias in a clearer way. The exact price of this option is 5.5616.
The Monte Carlo simulation for the soft barrier at a twenty-four hours monitoring frequency is shown in Figure 12.

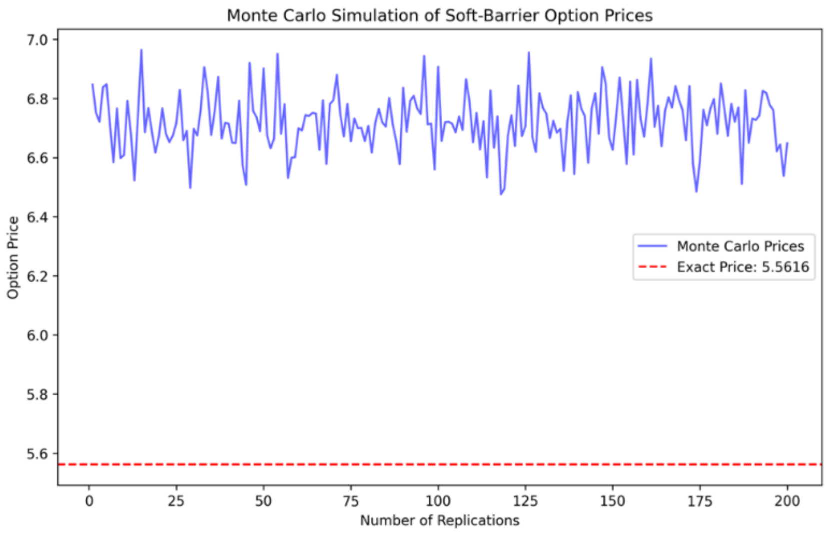


Figure 12: Monte Carlo Simulation for soft-barrier options prices, twenty-four-hour frequency monitoring.

The Monte Carlo simulation for the option at a one-hour monitoring frequency is shown in Figure 13.

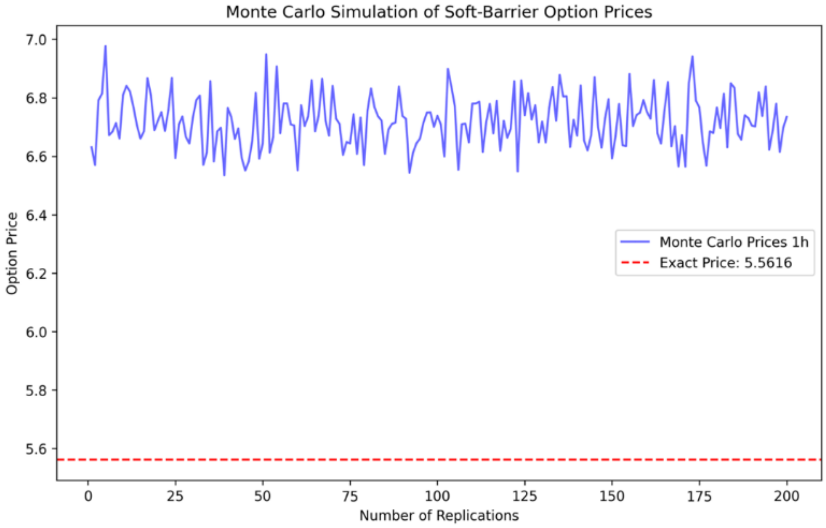


Figure 13: Monte Carlo Simulation for soft-barrier options prices, one-hour frequency monitoring.

The simulation results reveal a huge bias, similarly to the ones shown in the previous applications: as the monitoring frequency increases, though, the estimated price of the option does not increase as much as the other simulations, making it difficult to see the convergence toward the theoretical value that would be expected under a continuous monitoring.

3.3.2) Conditional Monte Carlo application

When we analyze this option, the bias resulting from the Crude Monte Carlo method is more evident, as it is subject to heavier numerical integration errors due to the inability to continuously monitor the underlying asset. As previously noted, the Babsiri-Noel's Conditional Monte Carlo method, is often preferred.
This approach proves particularly beneficial when the primary objective is to estimate the maximum or minimum values that the underlying asset may achieve over a specified time horizon.
By employing a numerical simulation, the method accurately tracks the asset price evolution while leveraging the properties of the Brownian Bridge. This mathematical construct enables the model to interpolate intermediate price points between two known values—typically the start and end points of the simulation period.
In doing so, the simulation can capture the probability distribution of the asset path more effectively, particularly when focusing on extreme movements.
As such, this technique provides a reliable way to assess the likelihood of the asset breaching certain levels within a predefined time frame, without the need for continuous monitoring, which would be computationally demanding. The Brownian Bridge framework ensures that even with discrete time steps, the method retains a high level of precision in determining the potential range of price fluctuations during the life of the option.

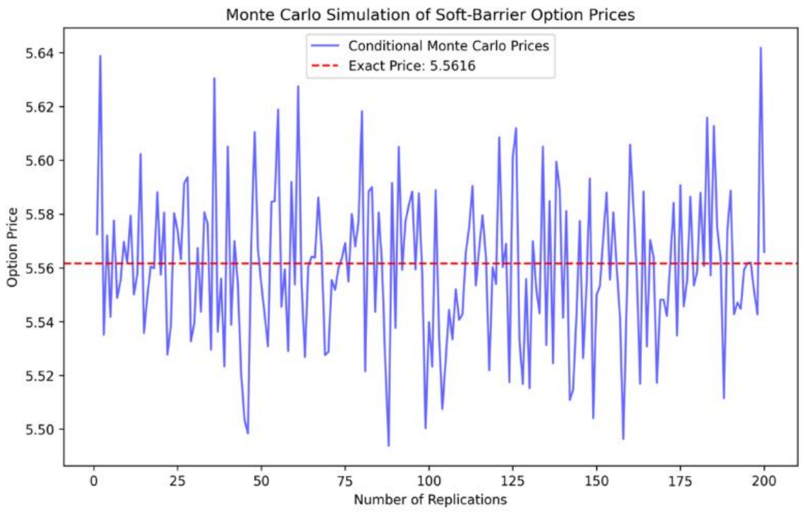
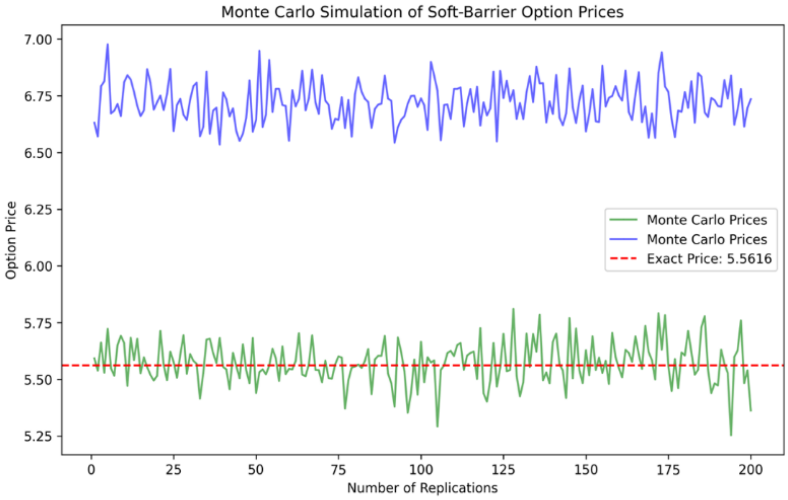


Figure 14: Conditional Monte Carlo simulation for Soft-Barrier Option prices

The Conditional method in Figure 14 shows the same advantages in both computational speed and precision. The Crude Monte Carlo method took increasingly longer with each reduction of the monitoring time window, in the Soft-Barrier case even longer than in the other cases, while the Conditional Monte Carlo method only took one minute, again.
By reducing the number of irrelevant paths, the Conditional method required far fewer simulations to reach a given level of accuracy. This reduction in computational effort translates directly into faster runtimes, making the CMC method the suitable choice here as well.
The plot is shown as follows instead of the histogram illustrated in the previous sections for the other kinds of options for the sake of clarity, being the results of the Crude Monte Carlo further than the results of the other cases.



	Mean	Std. Dev
24h	6.7202	0.0517
1h	6.7197	0.0469
30m	-	-
15m	-	-
Cond.	5.5895	0.0962

Figure 15: Comparison of Crude MC results with Conditional MC results.

3.4) Double Barrier Options

A double Barrier option is characterized by two barriers: one positioned above and one below the current stock price. It is classified as a path-dependent option, like the other barriers, as the holder's payoff is determined by the stock price interactions with these barriers. The contract specifies three distinct payoffs based on whether the stock price breaches the upper barrier, the lower barrier, or neither during the life of the option.

A barrier is a knock-out type if, upon being hit, the resulting payoff is a rebate (which may vary depending on the timing of the breach). Conversely, it is a knock-in type if hitting the barrier triggers a new option for the holder. The barrier feature can apply to the entire lifespan of the option or only to a portion of it.

A wide variety of double barrier options can be constructed to meet different risk management objectives by using different structural design. Similar to single barrier options, investors may use the exotic features of double barrier options to lower premiums, align with their expectations about future stock price movements, or meet specific hedging requirements.

Let τ_{up} and τ_{down} represent the first passage times at which the stock price breaches the upper and lower barriers, respectively, with T denoting the option maturity date. Double barrier options can be categorized based on the payoff structure, which depends on the relationship between τ_{up}, τ_{down} and T:

$\tau_{up} < \min(\tau_{down}, T)$:

This scenario arises when the upper barrier is breached before the lower barrier during the life of the option. For an up-barrier knock-in double barrier option, the holder receives a new option if the upper barrier is breached before the lower one. Otherwise, the option expires worthless.

$\tau_{up} < \tau_{down} < T$:

Here, the upper barrier is breached before the lower barrier within the life of the option. An example is the sequential double barrier option, where the option is knocked out when both the upper and lower barriers are breached in sequence. Essentially, once the upper barrier is hit, the option becomes a down-and-out single barrier option.

$\min(\tau_{up}, \tau_{down}) < T$:

This indicates that one of the barriers is breached before the maturity date. In a one-touch knock-out double barrier option, the option is knocked out if at least one barrier is hit, potentially resulting in a rebate payout.

$\max(\tau_{up}, \tau_{down}) < T$:

In this case, both the upper and lower barriers are breached within the life of the option. A double-touch knock-out option is knocked out only if both barriers are breached before maturity.

It is notable that combining an up-barrier knock-in option with an up-barrier knock-out option results in a standard European option. Similarly, a one-touch knock-in option can be split into an up-barrier knock-in and a down-barrier knock-in option. More complex payoff structures can be created based on the order in which barriers are breached. For instance, a double up-and-in call option expires worthless if neither barrier is breached during its life. If the upper barrier is breached first, the option transforms into a vanilla European call. However, if the lower barrier is hit first, the option becomes an up-and-in call option with a new upper barrier and strike price, effectively converting the option with adjusted parameters.

In the case of an occupation time derivative with double barriers, the payoff depends on the time that the stock price remains within a specified range or corridor. The option defines a corridor $[a, b]$ for the stock price, and if one of the barriers is breached, the option terminates, and the holder receives a payout proportional to the time the stock price spent within the corridor.

A double-barrier option is knocked either in or out if the underlying price touches the lower boundary L or the upper boundary U prior to expiration. The closed formulas shown below are for double knock-out options. The price of a double knock-in call is equal to the portfolio of a long standard call and a short double knock-out call, with identical strikes and time to expiration. In a similar way, a double knock-in put is equal to a long standard put and a short double knock-out put. Double-barrier options are priced with the Ikeda and Kuintomo closed formula (1992).

Call Up-and-Out-Down-and-Out:

Payoff: $c(S,U,L,T) = \max(S-K;0)$ if $L < S < U$ before T else 0.

$$c = Se^{(b-r)T} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n}\right)^{\mu_1} \left(\frac{L}{S}\right)^{\mu_2} [N(d_1) - N(d_2)] - \left(\frac{L^{n+1}}{U^n S}\right)^{\mu_3} [N(d_3) - N(d_4)] \right\}$$
$$- Ke^{-rT} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n}\right)^{\mu_1 - 2} \left(\frac{L}{S}\right)^{\mu_2} \cdot \left[N\left(d_1 - \sigma\sqrt{T}\right) - N\left(d_2 - \sigma\sqrt{T}\right)\right] \right.$$
$$\left. - \left(\frac{L^{n+1}}{UnS}\right)^{\mu_3 - 2} \left[N\left(d_3 - \sigma\sqrt{T}\right) - N\left(d_4 - \sigma\sqrt{T}\right)\right] \right\} \quad (108),$$

Where:

$$d_1 = \frac{\ln\left(\frac{SU^{2n}}{KL^{2n}}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (109)$$
$$d_2 = \frac{\ln\left(\frac{SU^{2n}}{FL^{2n}}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (110)$$
$$d_3 = \frac{\ln\left(\frac{L^{2n+2}}{KSU^{2n}}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (111)$$
$$d_4 = \frac{\ln\left(\frac{L^{2n+2}}{FSU^{2n}}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (112)$$

$$\mu_1 = \frac{2[b - \delta_2 - n(\delta_1 - \delta_2)]}{\sigma^2} + 1 \quad (113),$$
$$\mu_2 = 2n\frac{\delta_1 - \delta_2}{\sigma^2} \quad (114)$$
$$\mu_3 = \frac{2[b - \delta_2 + n(\delta_1 - \delta_2)]}{\sigma^2} + 1 \quad (115),$$

$$F = Ue^{\delta_1 T} \quad (116)$$

Where δ_1 and δ_2 determine the curvature L and U. The case of:

1. $\delta_1 = \delta_2 = 0$ corresponds to two flat boundaries.
2. $\delta_1 < 0 < \delta_2$ corresponds to a lower boundary exponentially growing as time elapses, while the upper boundary will be exponentially decreasing.
3. $\delta_1 > 0 > \delta_2$ corresponds to a convex downward lower boundary and a convex upward upper boundary.

Put Up-and-Out-Down-and-Out:

Payoff: $p(S,U,L,T) = \max(K-S;0)$ if $L < S < U$ before T else 0.

$$p = +Ke^{-rT} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n}\right)^{\mu_1 - 2} \left(\frac{L}{S}\right)^{\mu_2} \left[N\left(y_1 - \sigma\sqrt{T}\right) - N\left(y_2 - \sigma\sqrt{T}\right)\right] - \left(\frac{L^{n+1}}{U^n S}\right)^{\mu_3 - 2} \left[N\left(y_3 - \sigma\sqrt{T}\right) - N\left(y_4 - \sigma\sqrt{T}\right)\right] \right\}$$
$$- Se^{(b-r)T} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n}\right)^{\mu_1} \left(\frac{L}{S}\right)^{\mu_2} [N(y_1) - N(y_2)] - \left(\frac{L^{n+1}}{U^n S}\right)^{\mu_3} [N(y_3) - N(y_4)] \right\} \quad (117),$$

Where:

$$y_1 = \frac{\ln\left(\frac{SU^{2n}}{EL^{2n}}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (118)$$
$$y_2 = \frac{\ln\left(\frac{SU^{2n}}{KL^{2n}}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (119)$$
$$y_3 = \frac{\ln\left(\frac{L^{2n+2}}{ESU^{2n}}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (120)$$
$$y_4 = \frac{\ln\left(\frac{L^{2n+2}}{KSU^{2n}}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (121)$$
$$E = Le^{\delta_2 T} \quad (122)$$

The double-barrier options are expressed as infinite series of weighted normal distribution functions.

3.4.1) Crude Monte Carlo application

To model the presence of the two barriers, the Monte Carlo process involves simulating multiple potential future price paths for the underlying asset and checking, at each time step, whether the asset price has breached either of the two barriers.

For each simulated path, the algorithm tracks if and when the asset hits the upper or the lower barrier. If either barrier is crossed, the option is knocked out, and the payoff is set to zero. Conversely, if neither barrier is breached, the payoff is determined based on the specific option type (e.g., call or put).
The usage of the Crude Monte Carlo method is particularly effective for this application due to its ability to handle the multiple possible paths the asset price might take, especially under stochastic processes.
An Up-and-Out-Down-and-Out Call has been used for the simulation. The settings of the option price used to conduct the Monte Carlo simulations are as follows: $S = 100, K = 100, U = 150, L = 50, T = 0.25, r = 0.1, b = 0.1, \sigma = 0.35$.
Similarly to the simulation of the soft-barrier option, twenty-four hours, one hour and thirty minutes frequencies have been set: approaching from twenty-four hours to thirty minutes has shown very minimal improvements in the results of the simulations.
The number of simulations at each iteration is set to 10.000; the loop went for 200 iterations.
Each scenario was run through the Monte Carlo simulation to estimate the option price. As in the last section, the paths have been shown in the following graphs to show the bias in a clearer way. The exact price of this option is 7.0373.
The Monte Carlo simulation for the double barrier option at a twenty-four hours monitoring frequency is shown in Figure 16.

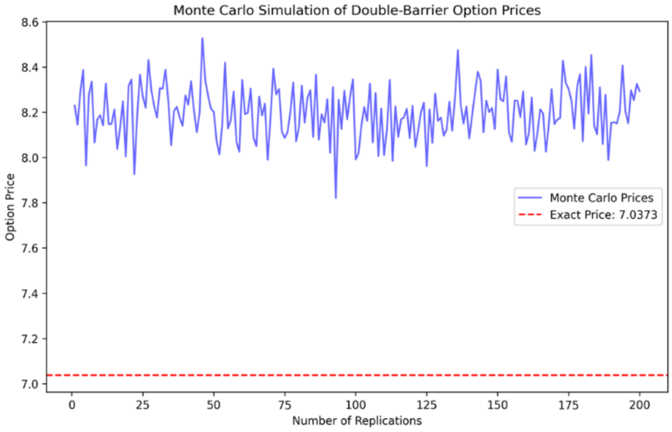


Figure 16: Crude Monte Carlo simulation at 24 hours for Double-Barrier Option prices

And the Monte Carlo simulation for the double barrier option at a one-hour monitoring frequency is shown in Figure 17.

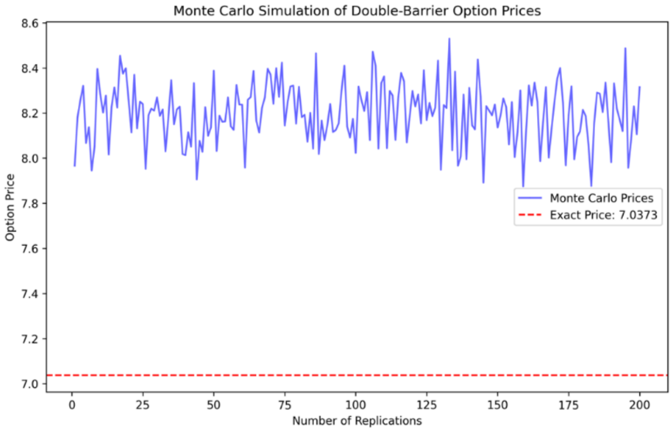


Figure 17: Crude Monte Carlo simulation at one hour monitoring frequency for Double-Barrier Option prices

In Figure 18, the simulation has a monitoring frequency of thirty minutes.

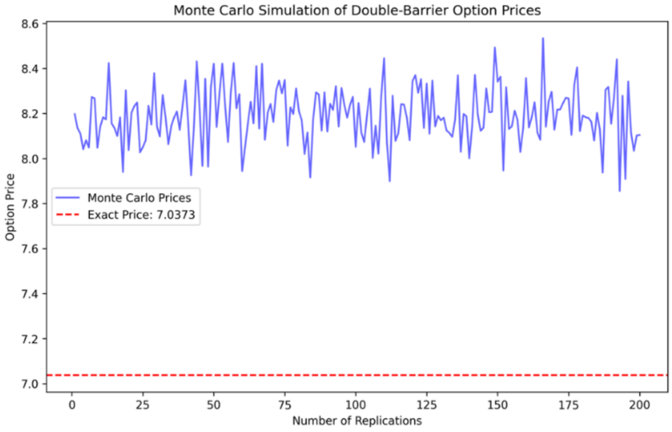


Figure 18: Crude Monte Carlo simulation at 30 minutes monitoring frequency for Double-Barrier Option prices

It can clearly be noted that not only is there a huge bias in all the three simulations, but the convergence to the exact value as the monitoring frequency increases is close to non-existent. The results can be defined as precise as the ones obtained in the soft-barrier option section, thus the different kind of plot rather than the histogram.

3.4.2) Conditional Monte Carlo application

As in the section dedicated to the soft-barrier options, the results obtained through the simulation of Crude Monte Carlo are non-optimal, even at narrower monitoring frequency time frames. The bias is too evident, and numerical integration errors due to the inability to continuously monitor the underlying are too heavy even at high frequencies. The Conditional Monte Carlo method is to be preferred here, too.
A simulation and an analysis are conducted on this kind of options, and the plots are presented below, shown in a similar way as the graphs illustrated in the soft-barrier section (see Figures 19 and 20).
First, a Conditional Monte Carlo simulation on the same option is shown, to illustrate the huge improvement on the calculation of the fair value. Then, comparison graphs are computed to show the difference between the Crude and the Conditional methods, for the sake of clarity and information.

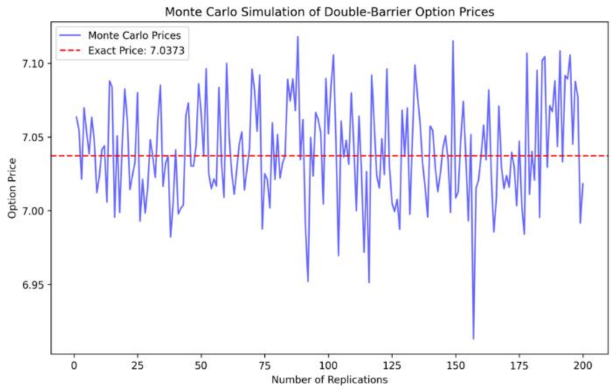
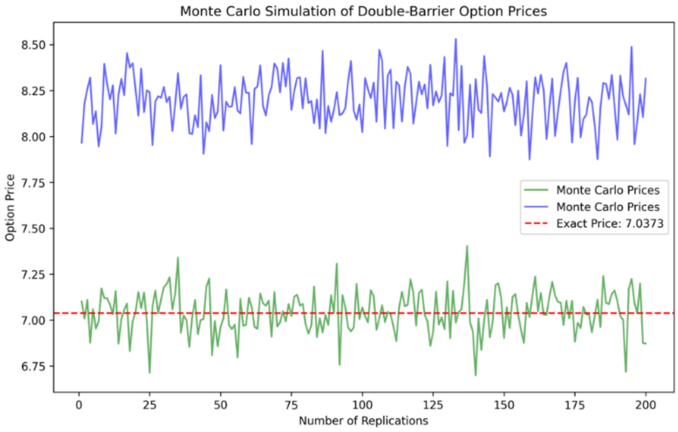


Figure 19: Conditional Monte Carlo simulation for Double-Barrier Option prices

As in the previous sections, the Conditional method in Figure 19 demonstrated the same advantages in both computational speed and precision.
The comparison between the Crude and the Conditional methods is illustrated, at 10.000 iterations each. The comparison is made on the thirty-minute monitoring frequency only, as the other graphs basically show the same divergence.



	Mean	Std. Dev
24h	8.2005	0.1160
1h	8.1981	0.1328
30m	8.1982	0.1328
15m	-	-
Cond.	7.0263	0.0962

Figure 20.a: Comparison between Crude and Conditional Monte Carlo methods for Double-Barrier Option prices

4) Market Case Study

In this section, the pricing of an investment certificate is implemented, highlighting that the Conditional Monte Carlo allows to obtain a fair value of the instrument unbiased and more aligned with market expectations. In order to perform this analysis, the structured product characterized by ISIN NLBNPIT1XYW7 issued by BNP Paribas was considered. All information is available both on the issuer's website and on the website of Borsa Italiana, so only the most important data used for pricing are reported here.

Issue Date: 22/12/2023

Exercise Date: 20/12/2024

Observation Period: from 22/12/2023 to 19/12/2024 (included)

Monitoring Style: American (i.e. Continuous time)

Currency: EUR

Notional Amount: EUR 100

Strike Price: EUR 6.668

Barrier Level: EUR 5.3344

Bonus Level: EUR 7,53484

Bonus Percentage: 113.00%

Cap Level: EUR 7.53484

Cap Percentage: 113.00%

Type of Settlement: Cash

The evaluation of the "Bonus Cap" product having Enel (IT0003128367) as underlying was conducted with the market data of February 16, 2024.

On the Settlement Date, the holder receives the following, for each certificate:

- If the Barrier Event has not occurred, a cash payment equal to the Notional Amount multiplied by the Bonus Percentage Level.

- Otherwise, if the Barrier Event has occurred, a cash payment equal to the lesser of (i) the Notional Amount multiplied by the Underlying Performance and (ii) the Notional Amount multiplied by the Percentage Cap Level.

The Underlying Performance is equal to the Underlying Reference Price divided by the Strike price. In such a case, the holder receives an amount less than the Notional Amount. Finally, the Barrier Event will be deemed to have occurred if the Price of the Underlying is at or below the Barrier Level at least once during the Observation Period.

The market data used for pricing are from info-provider Bloomberg, as of the valuation date. Figure 21 shows the interest rate term structure used for forwarding the underlying projections and discounting the terminal payment. The risk-free rate (r) has been interpolated considering the maturity of the product.

Curve # 45 - EUR (vs. 6M EURIBOR)
Curve Name EUR (vs. 6M EURIBOR)
Curve Side Mid
Swap Fixing EUR006M 3.89500 %
Interpolation Step Forward (Cont)
Curve Date 02/16/2024
DV01 Calc Type Shifting

Curve Data Shift +0.00 bp

Term	Market Ra...	Shift	Shifted Rate	Zero Rate	Discount
6 MO	3.89500	+0.00	3.89500	3.89808	0.980332
EUFR0A...	3.84800	+0.00	3.84800	3.78843	0.977729
EUFR0B...	3.72900	+0.00	3.72900	3.74018	0.974907
EUFR0C...	3.62800	+0.00	3.62800	3.70134	0.972203
EUFR0D...	3.50700	+0.00	3.50700	3.64658	0.969697
EUFR0E...	3.37750	+0.00	3.37750	3.60590	0.967064
EUFR0F...	3.26200	+0.00	3.26200	3.59073	0.964255
EUFR0I...	2.96000	+0.00	2.96000	3.41649	0.957946
EUFR01...	2.73100	+0.00	2.73100	3.31459	0.951195
2 YR	3.19025	+0.00	3.19025	3.13209	0.938877
3 YR	2.96875	+0.00	2.96875	2.91587	0.915728
4 YR	2.85450	+0.00	2.85450	2.80385	0.893495
5 YR	2.79325	+0.00	2.79325	2.74234	0.871475
6 YR	2.76000	+0.00	2.76000	2.71038	0.849533
7 YR	2.74900	+0.00	2.74900	2.70073	0.827377
8 YR	2.74545	+0.00	2.74545	2.69845	0.805478
9 YR	2.74565	+0.00	2.74565	2.69898	0.783879
10 YR	2.75000	+0.00	2.75000	2.70465	0.762629

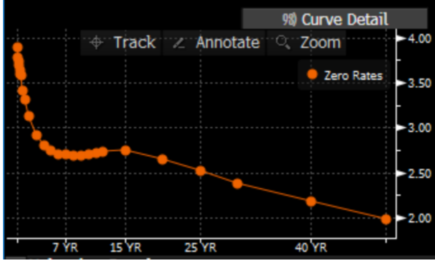


Figure 21: Interest rates term structure, tenor: 6 months. Source: Bloomberg®

Figure 22 shows the strip of implied dividend yields summarized from the call-put parity of actively traded options on the ENEL stock.

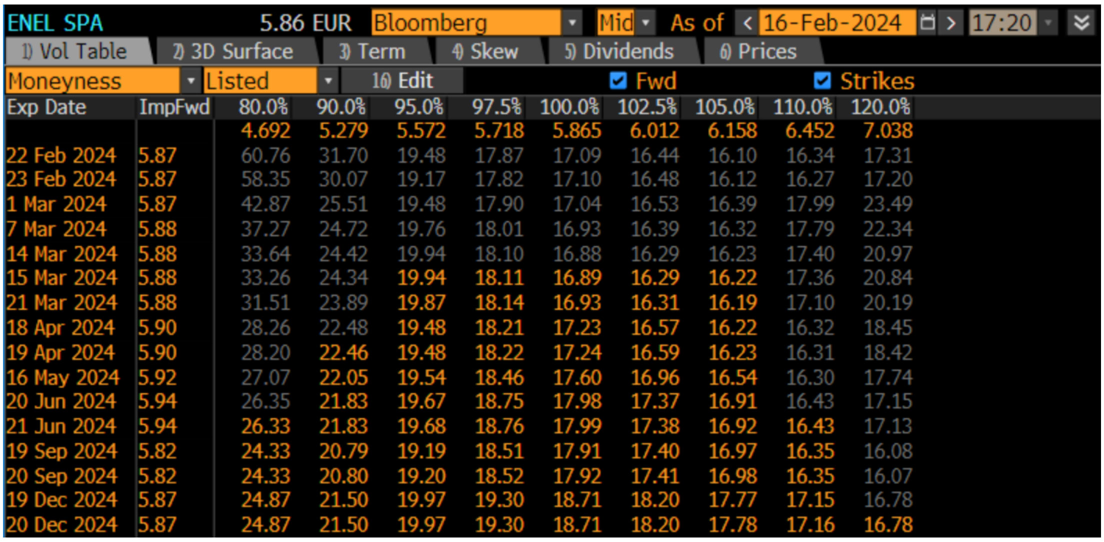
The dividend yield used (q) was interpolated by the time to maturity of the structured product. The reference spot price (S) for the calculations is the closing price on Feb. 16, 2024 (shown at the top left of the Figure).

ENEL IM Equity | 90) Asset | 91) Actions | 92) Views | 93) Settings
ENEL SPA 5.86 EUR Bloomberg Mid As of 16-Feb-2024 17:20
1) Vol Table | 2) 3D Surface | 3) Term | 4) Skew | 5) Dividends | 6) Prices
Listed Yields

Expiry	Exp Date	Impl Fwd	Risk Free	Impl Dvd	Impl (Yld)	BDVD Divs	BDVD (Yl...
22-Fe...	22 Feb 2024	5.87	3.358%	0.000	0.000%	0.000	0.000%
23-Fe...	23 Feb 2024	5.87	3.358%	0.000	0.000%	0.000	0.000%
1-Mar...	1 Mar 2024	5.87	3.358%	0.000	0.000%	0.000	0.000%
7-Mar...	7 Mar 2024	5.88	3.358%	0.000	0.000%	0.000	0.000%
14-Ma...	14 Mar 2024	5.88	3.358%	0.000	0.000%	0.000	0.000%
15-Ma...	15 Mar 2024	5.88	3.358%	0.000	0.000%	0.000	0.000%
21-Ma...	21 Mar 2024	5.88	3.430%	0.000	0.000%	0.000	0.000%
18-Ap...	18 Apr 2024	5.90	3.718%	0.000	0.000%	0.000	0.000%
19-Ap...	19 Apr 2024	5.90	3.724%	0.000	0.000%	0.000	0.000%
16-Ma...	16 May 2024	5.92	3.844%	0.000	0.000%	0.000	0.000%
20-Ju...	20 Jun 2024	5.94	3.840%	0.000	0.000%	0.000	0.000%
21-Ju...	21 Jun 2024	5.94	3.840%	0.000	0.000%	0.000	0.000%
19-Se...	19 Sep 2024	5.82	3.796%	0.177	5.112%	0.215	6.199%
20-Se...	20 Sep 2024	5.82	3.793%	0.177	5.088%	0.215	6.170%
19-De...	19 Dec 2024	5.87	3.656%	0.177	3.596%	0.215	4.361%
20-De...	20 Dec 2024	5.87	3.654%	0.177	3.585%	0.215	4.347%
19-Ju...	19 Jun 2025	5.77	3.390%	0.355	4.518%	0.430	5.476%

Figure 22: Implied Dividend Yield. Source: Bloomberg®

The implied volatilities surface is shown in Figures 23 and 24. The volatility (σ) has been interpolated considering the strike price and the time to maturity of the investment certificate.


ENEL SPA 5.86 EUR Bloomberg Mid As of 16-Feb-2024 17:20
1) Vol Table 2) 3D Surface 3) Term 4) Skew 5) Dividends 6) Prices
Moneyness Listed 10) Edit Fwd Strikes

Exp Date	ImpFwd	80.0%	90.0%	95.0%	97.5%	100.0%	102.5%	105.0%	110.0%	120.0%
		4.692	5.279	5.572	5.718	5.865	6.012	6.158	6.452	7.038
22 Feb 2024	5.87	60.76	31.70	19.48	17.87	17.09	16.44	16.10	16.34	17.31
23 Feb 2024	5.87	58.35	30.07	19.17	17.82	17.10	16.48	16.12	16.27	17.20
1 Mar 2024	5.87	42.87	25.51	19.48	17.90	17.04	16.53	16.39	17.99	23.49
7 Mar 2024	5.88	37.27	24.72	19.76	18.01	16.93	16.39	16.32	17.79	22.34
14 Mar 2024	5.88	33.64	24.42	19.94	18.10	16.88	16.29	16.23	17.40	20.97
15 Mar 2024	5.88	33.26	24.34	19.94	18.11	16.89	16.29	16.22	17.36	20.84
21 Mar 2024	5.88	31.51	23.89	19.87	18.14	16.93	16.31	16.19	17.10	20.19
18 Apr 2024	5.90	28.26	22.48	19.48	18.21	17.23	16.57	16.22	16.32	18.45
19 Apr 2024	5.90	28.20	22.46	19.48	18.22	17.24	16.59	16.23	16.31	18.42
16 May 2024	5.92	27.07	22.05	19.54	18.46	17.60	16.96	16.54	16.30	17.74
20 Jun 2024	5.94	26.35	21.83	19.67	18.75	17.98	17.37	16.91	16.43	17.15
21 Jun 2024	5.94	26.33	21.83	19.68	18.76	17.99	17.38	16.92	16.43	17.13
19 Sep 2024	5.82	24.33	20.79	19.19	18.51	17.91	17.40	16.97	16.35	16.08
20 Sep 2024	5.82	24.33	20.80	19.20	18.52	17.92	17.41	16.98	16.35	16.07
19 Dec 2024	5.87	24.87	21.50	19.97	19.30	18.71	18.20	17.77	17.15	16.78
20 Dec 2024	5.87	24.87	21.50	19.97	19.30	18.71	18.20	17.78	17.16	16.78

Figure 23: Implied Volatility Table. Source: Bloomberg®

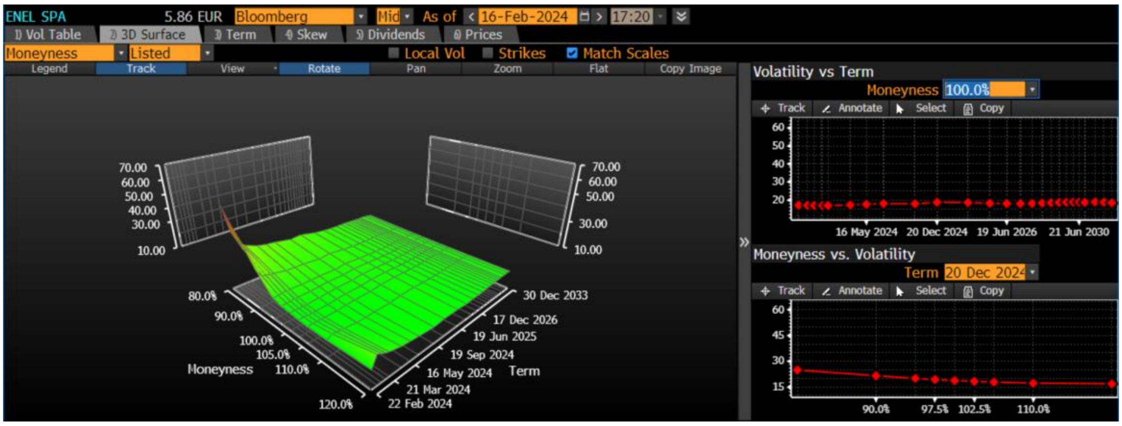


Figure 24: Implied Volatility Surface. Source: Bloomberg®

The spread to be applied to the discount factors so that the creditworthiness of the issuer is properly considered was derived from the one-year Senior CDS curve (Figure 25).

Curve Id	6M	1Y	2Y	3Y	5Y	7Y	10Y
11) CD353	18.6	21.5	26.9	32.2	43.9	56.0	70.9

Figure 25: BNP Paribas SA EUR Senior CDS Curve. Source: Bloomberg®

By evaluating the certificate using formulas that allow for the application of the Conditional Monte Carlo, a fair value aligned with market expectations at the analysis date is obtained (Figure 26). Specifically, the expected value achieved with 100 replications of 20,000 paths each is 93.70 ± 0.12. If a Quant were to estimate the price using the Crude Monte Carlo, the impact on pricing would lead to a distortion of up to 1.5.

Figure 26: Market quote for the certificate with ISIN: NLBNPIT1XYW7. Source: Borsa Italiana

5) Conclusions

This study highlights that the implementation of the Conditional Monte Carlo, if the underlying of the option follows a Geometric Brownian Motion and the financial instrument involves continuous monitoring of a threshold, entails two advantages: 1) the certainty of not introducing a numerical error resulting from an incorrect discretization of the motion; 2) the greater celerity in the processing of the pay-off by the calculation algorithm.
It is important to highlight that this methodology remains valid if we work under the assumption of valuating derivatives under the Black-Scholes-Merton pricing framework.
It is deemed interesting for the continuation of this study to verify the proper functioning of the methodology when applied to other second-generation options that involve continuous monitoring of a level and to quantify the evaluative bias accordingly. It would also be interesting to continue and analyze other investment certificates traded in the secondary market and characterized by a continuous monitoring. A proper valuation is also crucial, of course, for estimating sensitivity measures (Greeks), which constitute an essential tool for performing dynamic portfolio hedging.

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Enhancing IT Project Success Through Risk and Vulnerability Management: The Armenian Case

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Abstract

The research aimed to identify and evaluate the risks associated with IT projects, particularly focusing on their impacts. Despite numerous efforts, a significant number of software projects still fail to achieve success; however, these risks can be effectively managed. This study outlines methodologies for examining how different risks influence software projects, using statistical analyses and models to uncover causal relationships. A survey was also conducted to assess critical risk factors, highlighting three key factors that have the greatest influence. The findings suggest that addressing these factors can improve decision-making, thereby increasing the likelihood of project success.

Keywords: Armenian IT organizations, IT risk strategies, risk management, risk analysis, risk identification, risk monitoring, financial risks

1. Introduction

The purpose of this work is to identify both planned and unplanned risks encountered in the projects based on the experience of IT organizations and to analyze them. To achieve this goal, the following tasks were set:

• consider widely used methods for risk management in IT projects, including their pros and cons;

• collect data through a survey on the impact of risks on project performance in IT organizations;

• use statistical analysis to make a brief and meaningful summary of the main features of the database, analyze the differences between different categories of risks and the interrelationships between risks;

• divide the set of variables to be studied by means of factor analysis into a small number of groups, reducing the factors we have;

• use logistic analysis to find out which risks have a higher probability of impact on the success of the project;

• present summary conclusions and recommendations related to risks, management in IT projects.

The database for this article was formed by the data of a survey conducted among specialists of IT organizations, and the information basis included the researches, articles dedicated to IT projects by foreign authors of the IT sector, analyses conducted by international organizations, reports by the Project Management Institute (PMI), and completed reports and other materials.
Insights from foreign IT authors provide a global perspective, enriching the understanding of risk management practices. Analyses conducted by international organizations contribute to a comprehensive understanding of the challenges facing IT projects worldwide. Involving IT professionals, project managers, and organizational leaders in the research process provides a holistic view of the challenges and opportunities associated with risk management.
To pursue a comprehensive understanding of IT project risks, the research methodology employed in this study emphasizes advanced analytical tools. Table and graphic representation, descriptive, single factor (Anova: Single factor) and correlation analysis (Correlation) tools of MS-Excel software package, as well as factor analysis methods were used as research tools. The use of statistical techniques such as machine learning algorithms and predictive modeling enables more nuanced analysis of complex data sets. This methodological rigor goes beyond traditional approaches, providing a deeper understanding of the complex relationships between variables and the potential emergence of unpredictable risks.
This research seeks to provide actionable insights that can empower organizations to successfully face the complexities of modern projects. By examining widely used methods, surveying practitioners, and applying advanced statistical analyses, the study aims to contribute to a more nuanced understanding of risks in the ever-evolving technology landscape.
The expected results can serve as a practical guide for organizations, offering strategic recommendations for effective risk management in IT projects. In general, it can be concluded that the results along with the knowledge and tools can be useful for mitigating risk management problems in IT projects under the conditions of uncertainty. Integral to effective risk management is the ability to learn from experience, both successes and failures. Organizations that foster a culture of continuous learning and adaptation build resilience from the ground up. Each project becomes a repository of lessons, contributing to the organizational knowledge base. This approach not only strengthens risk management capabilities, but also fortifies the organization against future uncertainties.
The paper is organized into five sections. Section 1 introduces the background, aim, and significance of the study, while Section 2 provides a literature review on IT risk management, categorizing risks into pure and financial. Sections 3, 4, and 5 cover the research methodology, analysis and results, and the study's conclusions, respectively, highlighting key findings and their implications for IT project management.

2. Literature review

Risk management is a critical component of any organizational strategy, particularly in IT projects, where risks are dynamic and multifaceted. Unlike traditional forms of risk management that focus on general business or operational risks, IT risk management specifically addresses the uncertainties and challenges that arise from the use of information technology. It is essential to distinguish between pure risks, which involve the possibility of loss without any potential for gain, and financial risks, which can include both potential losses and gains depending on investment outcomes. This section explores the specificities of IT risk in the literature, with a focus on cybersecurity, data integrity, system downtime, and regulatory compliance, while addressing both pure and financial risks. IT risk, as defined by ISACA (2013), refers to the possibility that a given event or action could negatively impact the performance, security, or operational capacity of an organization's IT systems. IT risks are unique due to their rapidly changing nature, the complexity of IT infrastructures, and the broad range of potential threats, such as technological obsolescence, cyberattacks, and human error. These risks require tailored approaches that address both pure and financial risks, given the central role of IT in modern businesses.

Pure risks involve scenarios where only negative outcomes are possible, such as data breaches, system failures, or malware attacks. These risks typically require proactive management strategies aimed at preventing or minimizing potential damage. For example, the increasing frequency of cyberattacks has led to heightened attention to cybersecurity risks. Studies by Aven (2016) and Ponemon Institute (2020) highlight how breaches can lead to data loss, legal consequences, and reputational damage, all of which are forms of pure risk. To mitigate these risks, organizations are encouraged to implement strong security protocols, including firewalls, encryption, and continuous monitoring.

Financial risks, on the other hand, involve decisions where there is a chance of both loss and gain. For instance, investments in new IT systems, such as cloud infrastructure or advanced cybersecurity tools, carry financial risks. While the investment may lead to improved efficiency or enhanced security, there is also the potential for cost overruns or underperformance. McNeil et al. (2015) discuss how financial risk management tools, such as cost-benefit analysis, scenario planning, and sensitivity analysis can help organizations balance these risks, ensuring that investments in IT align with both the potential benefits and the risks involved.

One of the most critical aspects of IT risk management is cybersecurity, which is consistently ranked as a top concern for organizations (Deloitte, 2018). The literature emphasizes the evolving nature of cybersecurity threats, including phishing attacks, ransomware, and data breaches. A study by Westerman et al. (2014) shows how breaches can result in the loss of sensitive data, disruption of operations, and damage to customer trust. These are classic examples of pure risks in the IT domain—there are no potential gains from such events, only negative outcomes.

However, managing these risks often involves financial decisions, such as investing in cybersecurity solutions, hiring experts, or adopting cloud-based security services. In these cases, the financial risk of over-investment must be balanced with the pure risk of a breach, demonstrating the dual nature of risk in IT management.

The integrity and availability of data are also key risk areas in IT projects. Bezzina and Terribile (2019) highlight how issues, such as data corruption, accidental deletions, or unauthorized access can compromise the value and usability of critical business information. Downtime, whether caused by system failures or cyberattacks, can result in significant financial losses, especially in sectors that rely heavily on digital operations, such as finance and e-commerce (Henderson, 2017).

System downtime introduces a combination of pure and financial risks. Pure risks arise when downtime results in immediate losses, such as lost transactions or reduced customer satisfaction. Financial risks are present when organizations invest in preventative technologies such as redundant systems or disaster recovery plans, as these investments must be justified through potential cost savings or performance improvements (Schmidt and Altman, 2018).

As IT systems handle increasing amounts of personal and sensitive data, compliance with data protection regulations becomes a significant risk factor. Failure to comply with laws, such as the GDPR or HIPAA can lead to severe fines and legal action, creating a pure risk scenario. Choudhury and Vithal (2020) argue that organizations must not only protect data but also ensure that their systems and processes comply with relevant regulations. Compliance risks often lead to financial risks when organizations must invest in compliance measures, audits, or tools, such as encryption and data loss prevention (DLP) systems. These financial risks, while necessary, require careful planning and budgeting to ensure that they do not outweigh the benefits of regulatory compliance.

As highlighted, risk management in IT must address both pure risks (where the objective is loss prevention) and financial risks (where investments in IT infrastructure or risk mitigation are evaluated for potential gains and losses). This duality is essential in the IT field, where technology evolves rapidly and investments can quickly become outdated or ineffective. De Marco and Lister (2003) underscore the need for proactive risk management that distinguishes between these two types of risks, allowing for a balanced approach that mitigates losses while capitalizing on technological advancements.

The literature reveals that IT risk management involves complex, multi-dimensional risks that require both preventive and strategic financial planning. Effective management must consider the specificities of IT risks, such as cybersecurity, data integrity, system downtime, and compliance, while distinguishing between pure and financial risks. Organizations that implement comprehensive IT risk management strategies are better positioned to avoid negative outcomes while also leveraging opportunities for growth and innovation. In his research Dale Cooper emphasizes that risk management in projects is important for:

- managers, as it improves the basis for making appropriate decisions to meet operational requirements and achieve project objectives;
- the project staff, as it helps to identify things that can go wrong in the project process and suggests ways to solve them effectively;
- end-users, as it contributes to meeting needs and achieving value for money in the acquisition of key assets and capabilities;
- suppliers and contractors, because a sensible approach to risk in projects leads to better planning and better results for sellers as well as buyers;
- financiers who need to ensure that they receive a financial reward commensurate with the risks involved;
- insurers who require the comfort that risks are intelligently managed within the plan to determine how much to charge and whether to charge residual risk funding.

Risk management drives better business and project outcomes by providing insight, knowledge, and confidence to make better decisions. In particular, it supports better planning for contingencies, better allocation of resources to risks and alignment of project budgets, and better decision-making on the best allocation of risk among the parties involved in project activities. Together, they lead to increased certainty and reduced overall risk exposure. Risk management also provides a framework to avoid sudden surprises that

can be applied at all stages of the project cycle, starting from the earliest stages of evaluating the strategy for the supply, operation, maintenance, and disposal of individual items, facilities, or assets. Risk management will also provide benefits for better accountability and justification in decisions by providing a consistent process that supports decision-making.
During project implementation, the project team oversees all aspects, including risk management. In the article "Risk Management in Distributed IT Projects: Integrating Strategic, Tactical, and Operational Levels" that process is based on the CMMI model and includes 10 activities (Figure 1) aimed at simplifying and improving communication with stakeholders. It integrates the PMBOK Guide and MSF principles, starting with planning, identifying stakeholders, and adapting risk management strategies to align with organizational software development processes.
Among those activities, risk identification involves the project team and stakeholders looking for potential risks using planned techniques. It takes into account the project's requirements, assumptions, and constraints.
A standard list of risks based on previous projects can be used. These risks are then analyzed on a scale of 1 to 5 based on the likelihood and potential impact on project objectives.
The technical manager and the project manager work together to finalize the risk list. The fourth activity focuses on critical risk response planning, specifying response types, responsible parties, and timelines. The fifth activity involves following up on these planned responses and monitoring the probability of risk, and impact.
In the event of risk, unforeseen actions are taken, the control of which is defined in the sixth activity. Reporting of risk status (activity seven) takes place, which is reviewed by senior managers (activity eight).
After the project, the lessons learned are recorded in the risk database for future projects (the ninth activity). The 10th activity involves the review of the risks identified by the technical manager and the project manager.
By following this structured approach, project managers can proactively address risks, make informed decisions, and take the necessary actions to ensure project success.

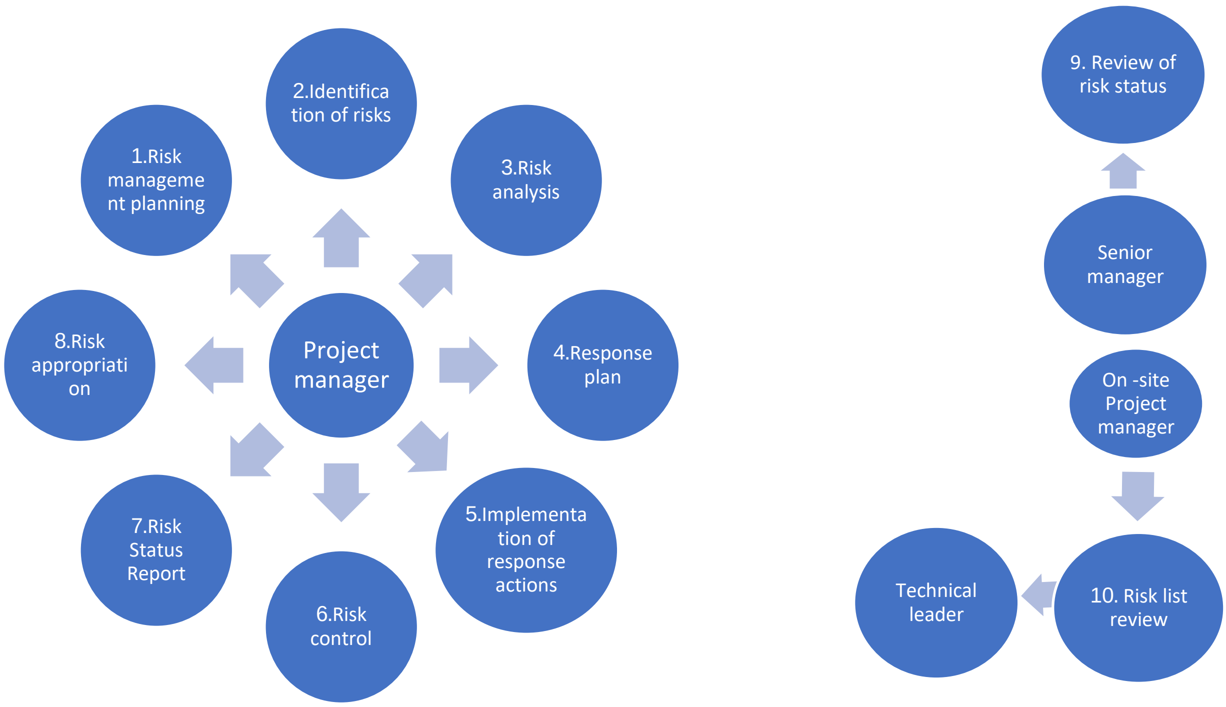

Figure 1. Project allocation to software design centers

Source: Rafael Prikladnicki, J. Roberto Evaristo, Jorge Luis Nicolas Audy, Marcelo Hideki Yamaguti: Risk Management in Distributed IT Projects: Integrating Strategic, Tactical, and Operational Levels, 2006

In their research, Bennett, Lienz, and Lee (2006) addressed the variety and complexity of common IT project risks, classifying them into three main types: internal issues and risks, external issues and risks, and issues and risks in specific IT activities.
Internal issues and risks refer to factors within the organization and project team.
These include team challenges, work being done, business units, governance, projects, and resistance to change. Controlling these problems is usually more feasible.
External issues and risks relate to external stakeholders and factors beyond the direct control of the IT team.
These include vendors, consultants, outsourcing, headquarters, international subsidiaries, technology, and business partners.
These issues are often more complex and political and may take longer to resolve.
Problems and risks in specific IT activities are associated with various phases of IT project life cycle, including analysis, software packages, development, implementation, and operations/support.
Each stage represents a distinct set of challenges.
In his study "Software Risk Management: Principles and Practices", Barry Boehm presents the top 10 software risk points and management techniques for each point:

Risk	Risk management techniques
Lack of personnel	Top talent recruitment, job matching, team building, training
Unrealistic timelines and budgets	Detailed cost and schedule estimation, design, incremental development, software reuse, requirements review, and refinement
Development of incorrect functions and properties	Organization analysis, mission analysis, operations formulation, user surveys and user participation, early user prototyping, quality factor analysis
Poor user interface development	Prototyping, scripts, task analysis, user participation
Over-engineering, adding features or elements	Requirements gathering, prototyping, cost-benefit analysis, cost-based design
A continuous flow of requirements changes	High change threshold, incremental development (postponing changes to later additions)
Defects in outfitted components	Benchmarking, checks, link checking, compatibility analysis
Deficiencies in task performance	Reference checking, pre-auditing, royalty contracts, competitive design or prototyping, team building
Real-time performance deficiencies	Modeling, benchmarking, prototyping, instrumentation
Computer science capability limitation	Technical analysis, cost-benefit analysis, prototyping, reference checking

Table 1. Top ten risks. Source: Barry Boehm "Software Risk Management: Principles and Practices" https://www.cs.virginia.edu/~sherriff/papers/Boehm%20-%201991.pdf, pp. 35

The author emphasizes a structured approach to risk mitigation techniques in IT projects, highlighting the importance of a top-10 risk tracking system. It outlines the following main points:

Risk resolution process. The process of mitigating risk in IT projects involves implementing strategies such as prototyping, simulation, benchmarking, and research per risk management plans.

Risk monitoring. Continuous monitoring of risk mitigation progress is critical to maintaining a closed process. It ensures that corrective actions are taken when needed to stay on track.

Tracking the top 10 risks. A critical aspect of risk management, this technique involves ranking the most important risks in a project and conducting regular reviews led by top management. The reviews focus on the top 10 risks, including their current ratings, history, and progress updates.

Centralization of management. By focusing management's attention on high-risk, high-leverage, and critical success factors, this approach saves time, reduces surprises, and enables managers to make a meaningful difference in project success.

Efficiency. The top-10 risk list ensures that management time is used effectively as it pinpoints issues where management intervention can be most effective.

Adaptability. The list can evolve with new concerns added and others removed based on their priority and progress, making it a dynamic and adaptive risk management tool.

In summary, the author advocates a structured, effective, and dynamic approach to reducing risk in IT projects by tracking top 10 risks, which keeps management focused on critical success factors and accelerates problem resolution.
Published by the Project Management Institute, *The Standard for Risk Management in Portfolios, Programs, and Projects* (2019) highlights the various techniques and methodologies used in IT project risk management, providing a comprehensive overview of the tools and processes used in the risk management lifecycle. It categorizes these techniques into three main types: templates and lists, process techniques, and quantitative techniques. These methods are designed to help identify, assess, and mitigate risks in IT projects. Risk management planning in the planning phase is very important to establish a common understanding of the risk approach and to document the risk management plan, which includes elements such as risk methodology, organization, roles, and communication plans.

Risk identification is a key step that includes techniques such as brainstorming, Delphi, interviews, historical data analysis, and SWOT analysis Tharanga, D. (2020). The book highlights the importance, threats, and opportunities of the methods.
Qualitative and quantitative risk analysis techniques help prioritize risks and provide a basis for resource allocation and response planning. Techniques such as affinity diagrams, probability and impact matrices, and sensitivity analysis play an important role at this stage.
Quantitative risk analysis aims to determine the overall risk for project objectives using methods such as decision tree analysis, expected monetary value (EMV) calculations, and Monte Carlo simulation.
In summary, risk management is an integral part of effective management, serving as the basis for achieving strong business and project outcomes, and effective procurement of goods and services.
Systematic risk identification, analysis, evaluation, and review of results significantly contribute to the success of projects. Researchers have developed a number of risk management methods in IT projects and different techniques and methodologies used in management, which can be selected to adapt to the needs, requirements and circumstances of the project. Risk should be considered in the earliest stages of project planning, and activities should continue throughout the project. Risk management plans and measures should be an integral part of the organization's management processes.
As IT systems become a critical competitive element in many industries, technology projects become larger, connecting more parts of the organization and putting the company at risk if something goes wrong. Unfortunately, projects often go wrong. Research by McKinsey with the University of Oxford shows that half of all large IT projects, defined as projects with an initial cost of more than $15 million, massively blow their budgets. On average, large IT projects are delivered 45 percent over budget and 7 percent over time, while delivering 56 percent less value than forecast. Software projects face the highest risks of cost and schedule overruns.
In a study of more than 5,400 IT projects by McKinsey and Oxford University's Center for Major Project Management, after comparing budgets, schedules, and projected performance benefits with actual costs and results, these IT projects were found to have a total of $66 billion in overruns, more than the GDP of Luxembourg Heygate (1994). It also found that the longer a project is planned to run, the more likely it is to run over time and budget, with each additional year spent on the project increasing cost overruns by 15 percent. Surveys of IT leaders have shown that the key to success is embracing four values that together make up the IT project methodology:

• a focus on strategy and stakeholder management instead of focusing solely on budget and planning

• assimilation of technology and project content

• building effective teams

• following key project management practices, such as strict quality checks.

Failure to master two of these typically accounts for almost half of the costs, while poor performance on the second two measures accounts for an additional 40 percent of the overhead.

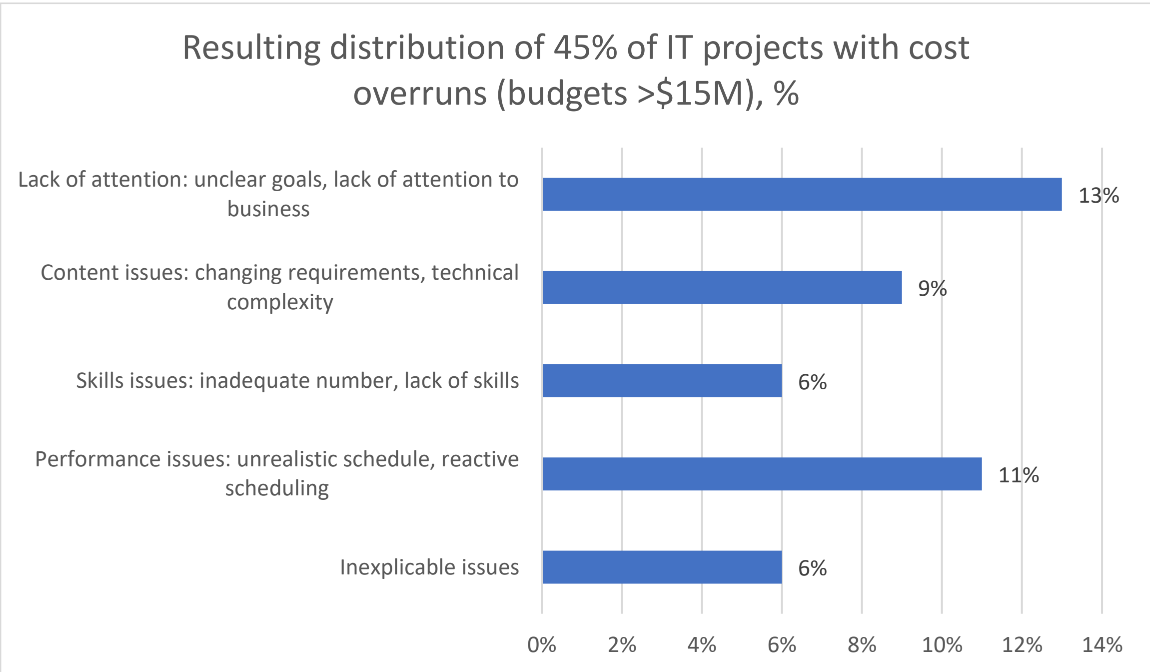

Figure 2: Four groups of problems identified by IT managers as causing most project failures

Source: https://www.mckinsey.com/capabilities/mckinsey-digital/our-insights/delivering-large-scale-it-projects-on-time-on-budget-and-on-value

The latest CHAOS study by the Standish Group, published in 2020, suggests a link between decision-making and project success. Teams with high decision-making skills deliver successful projects (63%) compared to skilled (28%), moderately skilled (20%), and non-skilled teams (18%) (Johnson, J. and Mulder, H., 2020).

Skill level	Successful	With challenges	Failed
With high skills	63%	30%	7%
With average skills	28%	61%	11%
With moderate skills	23%	51%	29%
With bad skills	18%	47%	35%

Table 2 Delaying decision skills. Source: Jim Johnson and Hans Mulder, 2021, "Endless Modernization: How Infinite Flow Keeps Software Fresh"

Over the past 25 years, the Standish Group has collected and studied 2,500 to 5,000 new project cases annually. Over those 25 years, they have added and changed observations to better understand why some projects succeed and others fail.

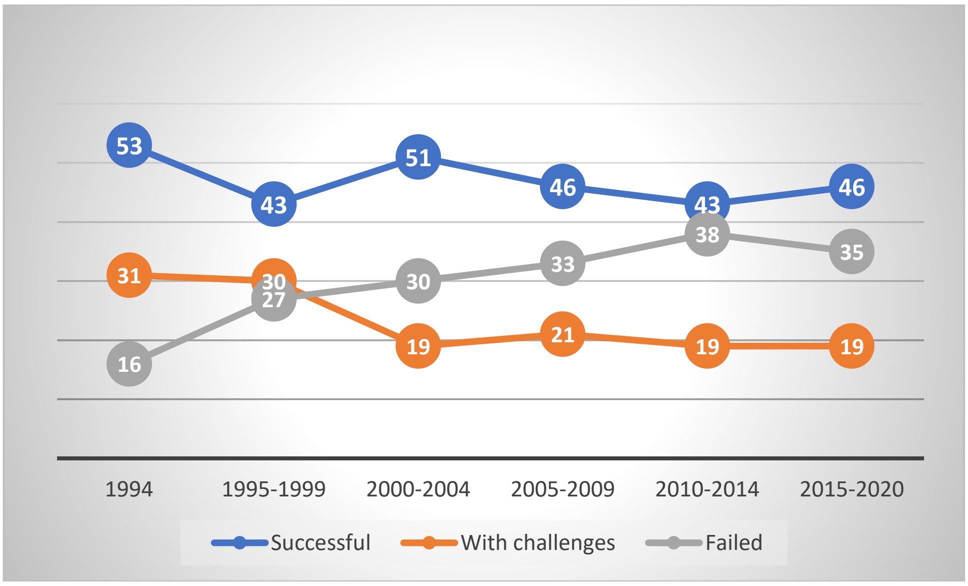

Figure 3: CHAOS survey

Source: Johnson, J. and Mulder, H., (2020). Endless modernization. Technical report, The Standish Group International, Incorporated.

3. Research methodoly

Risk assessments can be conducted to varying degrees of depth and detail using one or more different methods. Some common methods of risk identification include:

• Brainstorming method

• Delphi method

• SWOT analysis

• Root cause identification

Brainstorming is a technique for generating ideas among individuals or groups of people, where the ideas and thoughts of one individual serve to stimulate ideas among other participants. It is important to note that brainstorming belongs to the class of Synectics methods, which are not widely used due to their complexity. The key to this method is to carefully consider each idea. In practical application, the brainstorming method can encounter obstacles, because as a result, many ideas can be proposed that will be difficult or impossible to develop.

The Delphi method uses an anonymous survey of experts to identify risks. As a result, initial responses from experts are collected, subject to further analysis and generalization, and only then sent back to experts for review and further interpretation of risks based on the responses of others. This method allows you to analyze the risks several times, coordinate them, but one of the disadvantages is that it requires the participation of every member of the group, it takes a lot of time and is a heavy burden.

SWOT (Strength, Weakness, Opportunity and Threat) is a method that examines each aspect of SWOT to increase the breadth of risks being considered. This method focuses on internal (organizational strengths and weaknesses) and external (opportunities and threats) factors. The method has become very popular when conducting research in various business sectors, but one of the weaknesses of the SWOT analysis is the superficiality of the evaluated factors, only a qualitative description of the factors, and subjectivity. The conclusions drawn on its basis are descriptive without recommendations and priorities. The results of its implementation need additional analysis and methodological data, which will make it difficult to get an understanding in the field of risk management.

Root cause analysis helps identify additional dependent risks. Identified risks can be linked to their common root causes. The essence of this method is the detailed consideration of all possible risks, which are initially the result of certain activities and the creation of cause-and-effect relationships. One of the disadvantages of this method is the need for documentation, on which the identification, disclosure and analysis of risks can be based.

Taking into account the limitations of these methods the survey method was chosen for quantitative risk analysis. To conduct the survey, risks encountered in IT projects were grouped based on previous studies into the following categories:

• Project scope and requirements risks

• Resource risks

• Schedule risks

• Technological risks

• Communication risks

• Quality and compliance risks

• Security risks

For each category, four questions were included to assess the impact of the given risk on a scale of 1-5. This will allow for comparing the average impact within each category, as well as identify which categories have the highest and lowest average impacts.

The main objective of this survey is to study and evaluate the impact of various risks on project performance in IT organizations. To achieve this, the quantitative approach chosen allows for data collection, statistical analysis and generalization of results. The survey design is consistent with the research objective of assessing the multifaceted nature of risks within IT organizations. In this way, we can systematically collect quantitative data on respondents' perceptions.

This method is an effective way to reach geographically dispersed IT professionals, which is important to gain a wide range of perspectives, and the digital nature of the data it is based on provides clarity of interpretation. Additionally, given the dynamic nature of the IT industry, this approach allows us to assess current approaches and responses to risk.

To carry out the survey, IT organization specialists were selected as the target. Given the dynamic and multifaceted nature of IT project environments, a purposive sampling approach is most appropriate for this study. Purposive sampling allows you to choose those participants who have the necessary knowledge and experience in the field of risk management of IT projects and are familiar with the risks inherent in the projects. Selection criteria were developed to include a diverse range of IT professionals, including project managers, team members and stakeholders, to ensure a holistic perspective. To mitigate potential bias, the survey recruited participants from a variety of industries, project sizes, and geographic locations. This diversity minimizes the risk of skewed data, ensuring that the results are applicable to a wide spectrum of IT projects. In addition, anonymity and confidentiality were emphasized throughout the survey to encourage honest and unbiased responses. Fifty-seven participants were included in the survey due to practical constraints, such as time and budget. The study's narrow focus on specific IT project risk management measures provides a targeted approach, justifying the use of a smaller sample size. This sample size is adequate to capture key insights and patterns related to IT project risk management and allows for meaningful comparisons. This is presented in the attached Appendix 1. The survey was conducted online, which facilitated its effective dissemination among IT professionals and expedited the data collection process.

According to the obtained results, several steps were included for data analysis, providing a comprehensive analysis:

Data filtering and preparation: Before starting the analysis, the data collected during the research is filtered and prepared. This includes checking responses for completeness and accuracy. Missing or irregular data points are corrected to enhance the reliability of subsequent analyses.

4. Analysis

In this work, the analysis of the impact of risks of IT projects was carried out based on the data obtained from the results of the survey. There are seven factors in the database:

• Project scope and requirements risks

• Resource risks

• Schedule risks

• Technological risks

• Communication risks

• Quality and compliance risks

• Security risks

Analysis and construction of models were conducted using Excel and SPSS programs. To summarize and analyze the survey responses, descriptive statistics (Descriptive Statistics) were implemented in the work, to gain insight into various aspects of project management. Descriptive statistics provide a concise and meaningful summary of the key features of a database (Abbott, 2014).
By comparing mean scores, standard deviations, and other measures, we can identify areas of relatively higher or lower consensus. This helps prioritize areas that may require more attention in project management.

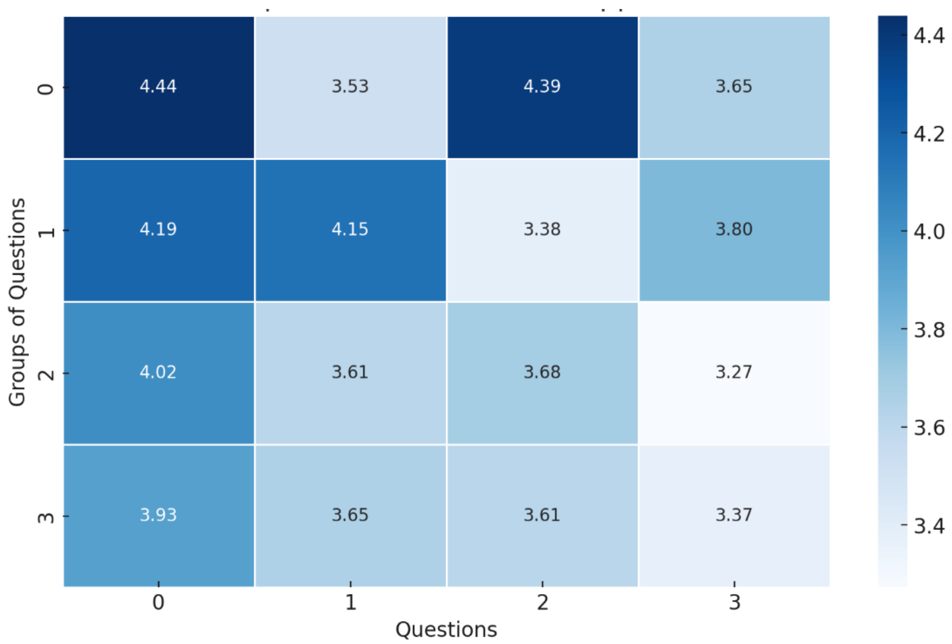

Figure 4. Heatmap of Mean Values from results of statistical description

Source: https://docs.google.com/spreadsheets/d/1iN0XTUMgNtdcN-u71HKxpIANs0-bCodf/edit?gid=766945934#gid=766945934

The figure 4 heatmap based on the mean values extracted from results of statistical description. The color gradient helps highlight the higher risk factors in darker shades, while lower values appear lighter, making it easier to visualize the significance of different risks. As we can see, the average value of insufficient or unclear program requirements of 4.44 suggests a high impact.
This can lead to project delays, cost overruns, and possible revisions, negatively impacting overall project performance. A mean of 3.53 for frequent project scope changes indicates a moderate effect. Although not as stringent as the unclear requirements, it carries the potential for increased costs and extended schedules. With a mean of 4.39 for ambiguous project objectives, this indicates a high impact. Uncertainty can lead to misunderstandings, affecting project implementation and increasing the likelihood of economic losses. A mean of 3.65 for alignment with stakeholder expectations indicates a moderate impact.
This can lead to discrepancies between project outcomes and stakeholder expectations, which can affect project success. A mean of 4.19 for underbudgeting indicates a high impact. This can lead to a lack of resources, affecting the quality of the project resulting in economic losses. Lack of necessary knowledge and skills: A mean of 4.15 indicates a high impact. Inadequate skills can lead to errors, delays, and cost overruns, adversely affecting project economics.
With a mean of 3.38 for resource constraints, there is a moderate effect. This can lead to challenges, but may not be as severe as budget-related risks. A 3.80 average for supplier or vendor-related risks allows for a moderate impact. Problems with suppliers or vendors can cause delays or cost overruns. A mean of 4.02 for Unreasonable Project Schedule suggests a high impact. Unrealistic schedules can lead to rushed work, errors, and increased costs. A mean of 3.68 for unplanned delays, a mean of 3.65 for technology compatibility issues, a mean of 3.61 for security vulnerabilities, and a mean of 3.37 for integration challenges indicate a moderate impact.
Meanwhile, a mean of 4.20 for poor communication between stakeholders and a mean of 4.16 for the risk of not meeting quality standards indicate a high impact. A mean risk of a security breach of 3.98 suggests moderate exposure.
Security breaches can lead to additional costs to address vulnerabilities and potential economic losses. Overall, the research findings highlight significant economic risks associated with various aspects of project management. Prioritizing risk mitigation strategies, ensuring effective communication, and allocating sufficient resources and budget are critical to minimizing economic losses and increasing project success.
Addressing areas such as insufficient budget allocation, poor communication, and security vulnerabilities should be a priority in project management strategies. Regular monitoring of project plans and adaptation to risk assessment are essential to successful project outcomes. It is important to note that these interpretations are based on statistical measurements and may not capture the full complexity of individual projects. The specific context of projects and industry standards must be taken into account when making strategic decisions.
In summary, the high-impact risks are insufficient budget allocation, unclear project requirements, unreasonable project schedules, poor communication, and security vulnerabilities. These areas require special attention because of their potential to significantly affect project outcomes and economic outcomes.
Moderate impact risks include resource constraints, dependence on external factors, technological challenges, and compliance issues. Although not as severe as high-impact risks, they also require active management to prevent negative consequences. In the next step, Anova's Excel program was applied to the database.

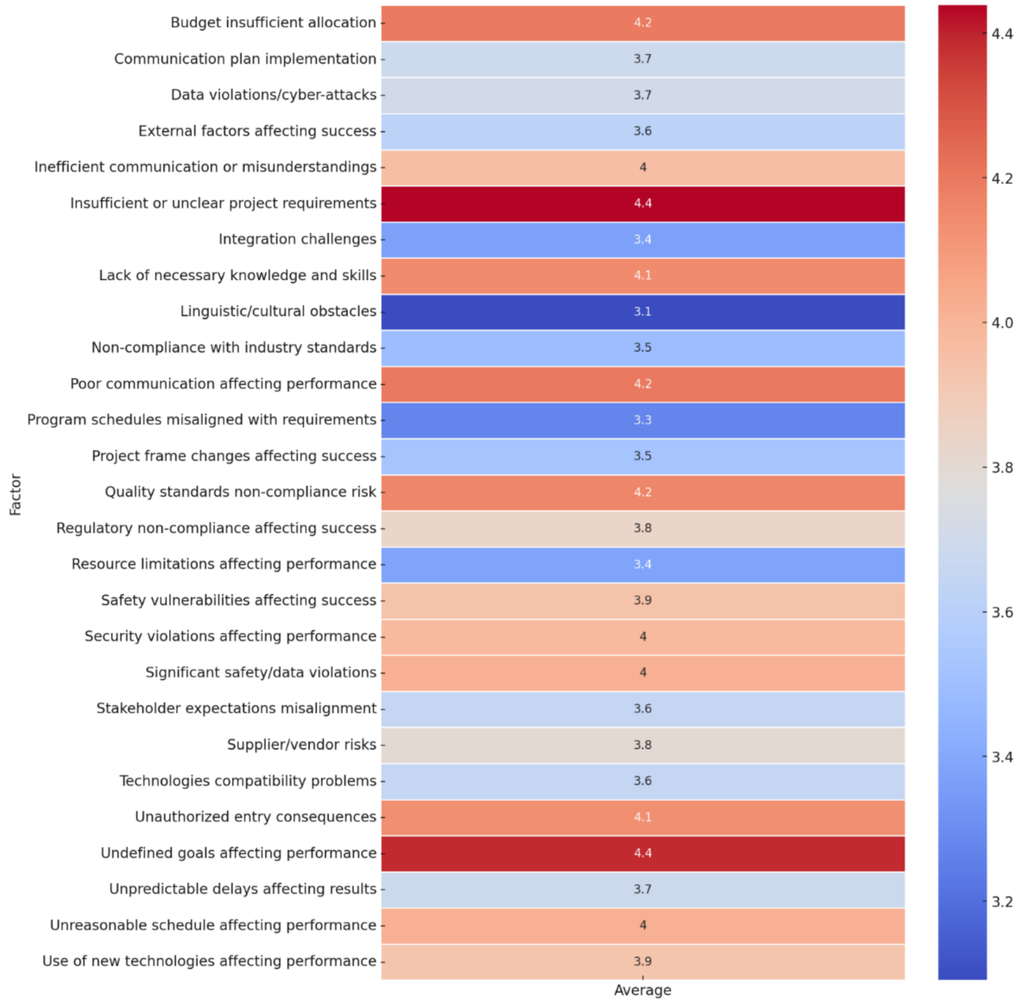

Figure 5. Heatmap of factors affecting project performances

Source: https://docs.google.com/spreadsheets/d/1iN0XTUMgNtdcN-u71HKxpIANs0-bCodf/edit?gid=1317595904#gid=1317595904

The figure 5 heatmap representing the average impact of various factors on project performance. The color intensity highlights the severity of each factor, with warmer colors indicating a higher average impact

The analysis shows significant differences between the different groups of the database, that is, the differences between the seven risk categories we identified, for each of which four questions were included. Hence, it is assumed that certain project risk factors differ significantly between different projects, regions, or divisions of the organization. Significant differences between groups suggest that these factors may have significant economic consequences. For example, in project management, the different risk levels of different projects can lead to significant differences in resource allocation, profitability, or the overall success of those projects. Understanding these significant differences can help better allocate resources. Economically, efficient allocation of resources based on these fluctuations can increase efficiency, minimize costs, and maximize revenues. For example, if some risk factors differ significantly between projects, prioritizing resource allocation based on those differences can optimize project outcomes. Significant variation between groups can also highlight areas with potential for improvement or growth. Economically, it can identify strengths or opportunities to reduce weaknesses in certain areas, leading to improved performance or market advantages. Recognizing significant differences between groups is important for risk management and investment strategies. Economically, this indicates the need for tailored risk mitigation approaches or targeted investment strategies based on these differences to optimize returns and minimize potential losses. In contexts outside of project risks, significant variation between groups may indicate different market segments or customer behavior. Understanding these differences can help target marketing strategies or customize services to meet specific customer needs, potentially increasing market penetration and revenue generation.

In general, the significance of variation between groups in ANOVA analysis has economic implications, and guides decision-making processes, resource allocation strategies, risk management, and opportunities for growth and market advantage in different segments or categories.

The resulting p-value of 1.1537E-23 is significantly decreasing and contradicts the null hypothesis. Such a p-value indicates, that in the context of the analysis, there are significant discrepancies between group means, which means significant variation within the variables studied. At the same time, the F-statistic of 6.822, which exceeds the critical F-value of 1.493, further supports the rejection of the null hypothesis. This statistical conclusion indicates a high level of confidence in confirming the existence of significant differences between the different groups included in the database. A significant difference observed between groups highlights different risk profiles, disparities in resource allocation or possible different market conditions among different project typologies or organizational segments. This difference may imply disproportionate resource utilization efficiency or distinct levels of risk exposure, thereby requiring specific strategic approaches to effective management and resource optimization. Furthermore, the statistical significance clarified by ANOVA analysis suggests the need for decision-making strategies. The statistical validation of significant differences highlights the importance of using these variations as potential avenues for growth and competitive advantage. Identifying and exploiting these differences can uncover hidden market opportunities, inform market penetration strategies, and facilitate tailored product and service offerings, thereby promoting market competitiveness and economic flexibility.

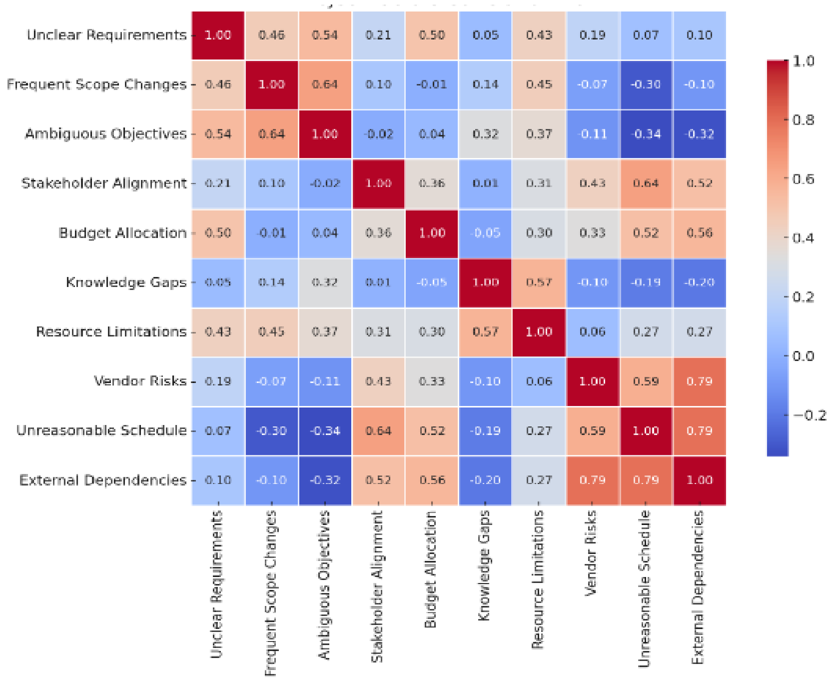

Figure 6. Project Factors Correlation Matrix

Source: https://docs.google.com/spreadsheets/d/1iN0XTUMgNtdcN-u71HKxpIANs0-bCodf/edit?gid=1366267207#gid=1366267207

The figure 6 heatmap based on the mean values. Here is the visual correlation matrix for the project-related factors. Each cell displays the correlation coefficient between pairs of factors, with color gradients indicating the strength and direction of the correlation.

Correlation analysis provides insights into the relationships between various project management factors. We find that insufficient or unclear project requirements are positively correlated with frequent project scope changes and ambiguous project objectives (Shi, H., et al., 2017). Stakeholder expectations may not align well with project objectives when requirements are unclear. Under-budget allocation is positively related to resource constraints and supplier or vendor-related risks. The lack of necessary knowledge and skills among team members has a weak positive correlation with resource constraints. Unreasonable project schedules are weakly positively associated with externalities and unanticipated delays. Project schedules may not match actual project requirements well, indicating potential scheduling challenges. The use of new technologies is positively correlated with technology compatibility issues, security vulnerabilities, and integration challenges. Technology-related factors have an impact on project performance, especially in terms of compatibility and security. Poor communication among project stakeholders is positively associated with ineffective communication among team members. Language or cultural barriers have a weak positive correlation with project outcomes (Han, P. C., 1996). Implementation of communication plans is positively correlated with effective communication. Failure to comply with industry standards is positively associated with failure to comply with regulatory requirements. Additionally, failure to meet regulatory requirements has a negative correlation with project success.

The risk of not meeting quality standards did not show a strong correlation with other factors in the analysis. Security breach risk has a weak negative correlation with project performance. Program security vulnerabilities do not show a strong correlation with the risk of security breaches. The consequences of unauthorized access to sensitive data or systems have a significant negative correlation with project success.

In summary, resource-related factors, including budget allocation and skills, are correlated with project outcomes.

The impact of IT project risks in the work was also investigated through factor analysis of multivariate statistics. Factor analysis is used to identify underlying factors that explain observed correlations between variables in data sets by dividing the set of variables under study into a small number of groups.

The following factors included in the survey were selected for multivariate analysis, the data on which are presented in Table 1:

1. To what extent do inadequate or unclear project requirements affect project performance (X1)?
2. To what extent do frequent project scope changes affect the success of your projects (X2)?
3. Please assess the impact of ambiguous project objectives on project performance (X3).
4. To what extent are the stakeholders' expectations consistent with the project's goals (X4)?
5. How does insufficient budget allocation affect project performance (X5)?
6. Please assess the impact of the lack of necessary knowledge and skills of team members on the success of the project (X6).
7. To what extent do resource limitations (hardware, software, tools) affect project results (X7)?
8. How significant are supplier or vendor-related risks in your projects (X8)?
9. Please assess the impact of unreasonable project schedule on project performance (X9)?
10. To what extent does dependence on external factors affect the success of your project (X10)?

11. To what extent do unforeseen delays affect project results (X11)?
12. To what extent are project schedules consistent with actual project requirements (X12)?
13. How does the use of new technologies affect the implementation of the project (X13)?
14. Please assess the impact of technology compatibility issues on project success (X14).
15. To what extent do security vulnerabilities affect the success of your projects (X15)?
16. How significant are integration challenges in your projects (X16)?
17. How does poor communication between project stakeholders affect project performance (X17)?
18. Please evaluate the impact of ineffective team member communication or misunderstanding on project success (X18).
19. To what extent do language or cultural barriers affect the results of your project (X19)?
20. How well is the communication plan followed and implemented in your programs (X20)?
21. How does the risk of non-compliance with industry standards affect project performance (X21)?
22. Please assess the impact of non-compliance with regulatory requirements on project success (X22).
23. To what extent does the risk of not meeting quality standards affect the results of your project (X23)?
24. How significant are the consequences of security or data breaches due to non-compliance with security standards or regulations in your projects (X24)?
25. How does the risk of security breaches affect project performance (X25)?
26. Please assess the impact of project security vulnerabilities on project success (X26).
27. To what extent do data breaches or cyber-attacks affect the results of your project (X27)?
28. How significant are the consequences of unauthorized access to sensitive data or systems in your projects (X28)?

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	8.014	28.621	28.621	8.014	28.621	28.621	4.370	15.606	15.606
2	4.655	16.625	45.247	4.655	16.625	45.247	4.235	15.125	30.732
3	4.277	15.273	60.520	4.277	15.273	60.520	4.183	14.939	45.671
4	2.022	7.222	67.742	2.022	7.222	67.742	3.400	12.143	57.814
5	1.545	5.519	73.261	1.545	5.519	73.261	2.716	9.700	67.514
6	1.444	5.158	78.418	1.444	5.158	78.418	2.387	8.524	76.038
7	1.151	4.110	82.528	1.151	4.110	82.528	1.817	6.491	82.528
8	.904	3.230	85.759						
9	.859	3.068	88.827						
10	.651	2.324	91.150						
11	.551	1.967	93.117						
12	.449	1.605	94.723						
13	.398	1.420	96.143						
14	.282	1.008	97.152						
15	.222	.792	97.944						
16	.207	.741	98.684						
17	.139	.496	99.180						
18	.091	.326	99.506						
19	.073	.261	99.767						
20	.026	.094	99.861						
21	.023	.081	99.942						
22	.010	.037	99.979						
23	.005	.019	99.999						
24	.000	.001	100.000						
25	1.088E-15	3.887E-15	100.000						
26	1.921E-17	6.860E-17	100.000						
27	-1.274E-16	-4.551E-16	100.000						
28	-3.813E-16	-1.362E-15	100.000						

Table 3. Total Variance Explained. The factor analysis was carried out using the SPSS software package.

Source: https://docs.google.com/spreadsheets/d/1iN0XTUMgNtdcN-u71HKxpIANs0-bCodf/edit?gid=1102970142#gid=1102970142

Accordingly, three factors were selected for analysis. The first explains 28.621% of the total variance, the second explains 16.625%, the third explains 15.273%, and the three factors together explain 60.6% of the total variance (Table 3).
The next step in interpreting the results of the factor analysis is to look at the rotated component matrix of the factor coefficients. This table is the main result of the factor analysis, in which the results of the classification of variables by factors are expressed.
As can be seen from Table 4, the 13 studied variables were classified according to three factors: 5 variables can be included in the first one, 5 variables in the second one, and 3 variables in the third one.

	Component		
	1	2	3
X_1	-.143	.204	.056
X_2	-.426	.178	-.070
X_3	-.133	.139	-.425
X_4	.224	.115	.687
X_5	.376	.092	.430
X_6	.225	.818	-.276
X_7	-.245	.814	.226
X_8	.045	.053	.491
X_9	.041	-.264	.439
X_{10}	.214	.018	.769
X_{11}	.058	.110	.878
X_{12}	-.192	.111	.030
X_{13}	.047	.408	.408
X_{14}	.076	.807	.277
X_{15}	.452	.447	.040
X_{16}	.242	.280	-.014
X_{17}	.170	.671	-.019
X_{18}	.208	.623	-.118
X_{19}	-.167	.293	-.045
X_{20}	-.233	.347	.251
X_{21}	.401	-.019	.316
X_{22}	.203	-.058	.416
X_{23}	.002	.115	.296
X_{24}	.728	-.154	.296
X_{25}	.710	.480	.111
X_{26}	.860	.285	-.038
X_{27}	.713	.123	.071
X_{28}	.864	-.108	.121

Table 4 Rotated Component Matrix.

The table was created by authors using SPSS software. Extraction Method: Principal Component Analysis. Rotation Method: Varimax with Kaiser Normalization.

We tentatively named the first factor "**Project Security Risks**", because it includes the risk of compatibility violations (0.728), the risk of unauthorized access (0.71), the risk of cyber security threats (0.86), the risk of cyber security incidents (0.713), the risk of consequences of unauthorized access (0.864).
The second factor was named "**Project Internal Risks**" because it includes the risk of lack of necessary knowledge and skills of team members (0.82), the risk of resource limitations (0.81), technology compatibility issues (0.807), the risk of communication between stakeholders and (0.671), risk of gaps in team communication (0.623).
The third factor named "**External Risks**" includes three variables: risk of alignment of expectations (0.68), risk of external dependence (0.76), and risk of delay effect (0.879).

Reliability analyses.

According to Cronbach's test, the tabular value was obtained as 0.887, which means that the alpha values of the factors have acceptable values (exceeding 0.5), therefore the data are reliable for conducting analysis.
Thus, since the reliability of the data is checked, the following hypotheses are proposed in the work:

• **Hypothesis 1**. Project security risks are affected by dynamic project factors, such as technological advances, regulatory changes, and human behavior.
• **Hypothesis 2**. Internal project risks affect outcomes.
• **Hypothesis 3**. External risks affect project performance.

To test these hypotheses, the logistic regression model was employed.

The investigated logistic model has the following form:

$$P(Y = 1|X = (X_1, X_2, X_3)) = \frac{e^{\tilde{Y}}}{1 + e^{\tilde{Y}}} = \frac{1}{1 + e^{-\tilde{Y}}}$$

Where:
In our example, the linear regression equation looks like this:

$$\tilde{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

This equation gives the probability that one outcome $Y = 1$ based on the predictors $X_{1,} X_2$ and X_3.
Where:

$$\text{Project success depends on risk management: } Y_i = \begin{cases} 0, & if\ the\ event\ does\ not\ occur \\ 1, & if\ the\ event\ occurs \end{cases}$$

Project security risks: X_1
Project implementation risks: X_2
Risks of external dependence: X_3
Unknown model parameters: β_0, β_1, and β_3.

Therefore, a logistic regression analysis is performed between the factors of the dependent variable (project success depending on risk management) and the independent variables (project security risks, project internal risks, and external risks).

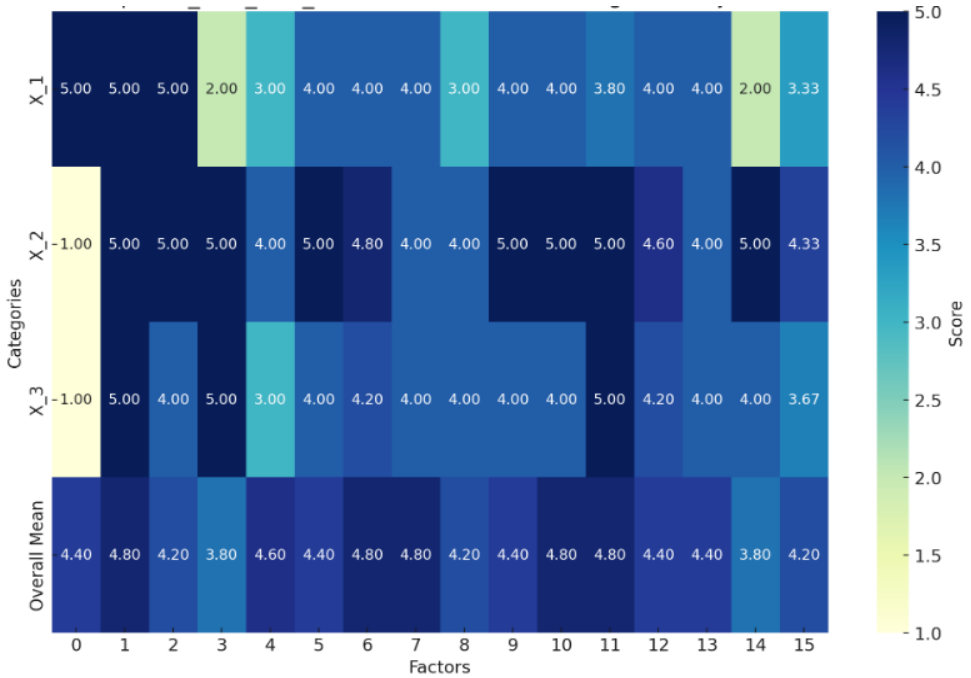

Figure 7. X_1, X_2, X_3 overall mean scores overall average ratings of factors

Source: https://docs.google.com/spreadsheets/d/1iN0XTUMgNtdcN-u71HKxpIANs0-bCodf/edit?gid=1366267207#gid=1366267207

Here is the heatmap of the overall mean ratings for X_1 , X_2 , X_3 and the general average ratings of project factors. Each row corresponds to one of these categories, with the color intensity reflecting the score values across different factors.
The relationship between the dependent variable and the independent variables is checked by applying the Omnibus Tests of Model Coefficients, the results of which are shown in Table 5. The Chi-Square value of the model is 46.731, and the p-value is less than 0.05, which means that our model is highly significant (Table 5).

		Chi-square	Df	Sig.
Step 1	Step	46.731	3	.000
	Block	46.731	3	.000
	Model	46.731	3	.000

Table 5 A test of coefficients for the Omnibus Model. The table was created by authors using SPSS software

In logistic regression, to determine multicollinearity between independent variables, numerical errors must be detected and problematic variables must be excluded from the analysis. Therefore, the standard error (SE) column of the variables in the equation table is checked if there is any value above 2.0. Thus, we can conclude that there is no problem with the variables being significantly dependent on each other, as the SE values in the table are below 2.0.

		B	S.E.	Wald	Df	Sig.	Exp(B)
Step 1[a]	X1	.424	.397	1.137	1	.007	1.231
	X2	.388	.355	1.199	1	.005	1.474
	X3	.271	.377	.515	1	.008	1.654
	Constant	.264	1.929	.019	1	.009	.768

a. Variable(s) entered on step 1: X1, X2, X3.

Table 6. Variables in Equations. The table was created by authors using SPSS software

From the obtained results, it can be seen that the p-value for the project safety risks factor is (0.007), for the internal risks factor (0.005), and for external risks is (0.007), which are less than 0.05 and also less than 0.01, which means that these independent variables are statistically significant (Table 6).
According to the final results, we have the following picture:
Hypothesis 1: Project security risks are affected by dynamic project factors such as technological progress, regulatory changes, and human behavior, the hypothesis is accepted.
Hypothesis 2: The hypothesis that internal risks of the project affect the results is accepted.
Hypothesis 3: The hypothesis that external dependency risks affect project performance is accepted.
The value of Exp (B) for the risk factor of external dependence is 1.654, which means that for each one-degree increase in external dependence on the rating scale, the probability of impact on project performance increases by 1.654 times. In other words, for every one-step increase in the risk level of external dependence, the probability of impact on project performance increases by 65 percent over the previous step. It is observed that there is a highly positive relationship between external dependency risks and the impact on project performance.

The value of Exp (B) for the project safety risk factor is 1.231, which means that increasing the risk by one degree increases the probability of impact on the dynamic factors of the project by 1.231 times. With each degree of increase in the level of security risks of the project, the probability of impact on the dynamic factors of the project increases by 23 percent compared to the previous degree. Thus, it is observed that there is a positive relationship between project security risks and the impact on project dynamic factors.

The value of Exp (B) for the internal risk factor of the project is 1.474, which means that increasing the risk by one degree increases the probability of impact on the results by 1.474 times. With each degree of increase in the level of internal risks of the project, the probability of impact on the dynamic factors of the project increases by 47 percent compared to the previous degree. Thus, it is also observed that there is a positive relationship between internal project risks and their impact on outcomes.
Based on the results of logistic regression, and the results of parameter estimation to extract the probabilities of different situations, we will have the following regression equation:

$$\ln\left(\frac{P}{1-P}\right) = \hat{Y}$$

$$\hat{Y} = 0.264 + 0.424 * 4.4 + 0.388 * 4.8 + 0.271 * 4.33 = 5.154$$

$$p = \frac{1}{1 + e^{-5.154}} = 0.89425692$$

In other words, if the impact of security risks, internal risks, and external risks in the IT organization is rated as high, the success of the project, depending on risk management, is 0.894 or 89.4%.

5. Conclusion

The significance of risk management has grown remarkably for contemporary organizations, as risks can lead to both detrimental losses and potential opportunities. A robust risk management framework can effectively minimize and avert risks. This research delved into the theoretical foundations of risk and essential risk management practices, emphasizing risk identification as a pivotal phase. Various risk assessment techniques were analyzed, each presenting distinct advantages and disadvantages. The findings from the survey underscored critical risks associated with project management, such as cybersecurity vulnerabilities, unauthorized access, skill shortages, resource constraints, and communication deficiencies. A correlation analysis revealed that effective communication and the adoption of technology are essential for the success of projects. Furthermore, logistic analysis demonstrated that external dependencies, security concerns, and internal risks have a substantial influence on project outcomes. To lessen these risks, numerous strategies have been suggested, namely the adoption of strong encryption techniques, the implementation of multi-factor authentication, conducting regular security audits, providing employee training programs, formulating incident response plans, and building a strong sense of security awareness throughout the companies. Moreover, it is advisable to perform geopolitical risk evaluations, remain informed regarding regulatory developments, and mitigate dependence on individual external entities to alleviate external risks. These strategies are designed to foster a secure and resilient environment for IT projects, thereby enhancing their likelihood of success.

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Sustainable Investments and ESG factors

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1. Introduction

In recent years the transition towards a model of sustainable economic development has assumed central importance for the financial system. The inclusion of environmental, social and governance aspects (ESG – Environmental, Social and Governance) is taking an important role in the investment and issuance process, encouraging innovation and the growth of sustainable finance, which sees the application of the concept of sustainable development to financial activities.

The energy crisis and the Russia-Ukraine war are two factors that can contribute to accelerating the energy transition to reduce dependence on Russian gas imports and more generally on fossil fuels. The war resulted in rising energy prices, further pushing European countries to reduce their dependence on Russian oil and gas supplies.
Regulators are increasingly focused on transparency regarding sustainable investments. Financing the transition to a low-carbon economy is crucial today, given the impact that climate change continues to have on economies, businesses and communities globally. To finance decarbonisation across sectors, innovative solutions will be needed, the development of which will require large amounts of capital. In this context, the sustainable bond market has become of considerable importance to find the financial resources necessary to fill a financial gap of 4,100 billion dollars by 2050[2] .
ESG bond issues (Green bonds, Social bonds and Sustainability bonds) have shown an exponential increase in recent years. This growth has drawn impetus from the indications of the European Action Plan (2018) and the EU Green Deal 2019, which "given the insufficiency of public funds" aim to "fill the financing gap through the mobilization of private capital". Sustainable bonds can therefore represent a useful tool to achieve this target. ESG factors play an important role in their investment decisions. Regulatory authorities, rating agencies and the stakeholders' globally are showing a growing interest in ESG issues, leading to new requirements in measurement and management processes and increased reporting needs. This constant flow of new regulations is bringing new compliance challenges to banks.
The aim of this work is to highlight the importance that ESG (Environmental, Social and Governance) factors have on the economic system. Sustainability is an increasingly relevant topic and factors related to the environment, sustainability and governance have become increasingly important to investors, who use corporate social responsibility scores as a guide to avoid high financial risks or questionable business practices.

The integration of ESG criteria in the financial sector is taking on an increasingly important role. In a context where environmental and social concerns are gaining more and increasing attention, investing sustainably has become a key objective for many investors. ESG factors allow for greater knowledge of financial risks: financial and non-financial companies that do not adequately manage their environmental impacts are exposed to greater risks. Companies with a strong ESG profile are less vulnerable to systematic market shocks and therefore have lower systematic risk. The challenge of sustainable development gives ESG factors an increasingly important role in evaluating investment opportunities and risks. The concentration of a part of the portfolio in instruments built in compliance with ESG standards provides a natural protection to the challenge of the green transition of the economy.
In impact investing, investors not only seek to obtain a financial return by optimizing risks, but also set objectives linked to the social and environmental impact that companies aim to achieve. This approach assumes that companies must pursue broader objectives than just generating economic value. According to many economists and observers, an evolution is underway that aims to overcome the idea that shareholders'well-being is limited to profits and growth of market value. It is considered, however, that activities aimed shareholders' well-being generating profits and those inspired by ethical principles are not inconsistent, but, on the contrary, are destined to become inseparable, especially for investors who adopt a long-term perspective.
Companies, banks, states, other public bodies and supranational bodies that issue green bonds to attract new investors, in addition to possibly reducing the cost of financing, have a positive impact in terms of image.

2. ESG risks - New EBA guidelines

EU countries have committed to achieving the goal of climate neutrality by 2050 by meeting their commitments under the Paris Agreement. The European Green Deal is the EU's strategy to achieve the goal by 2050.

The impacts of physical events and the necessary transition to a low-carbon, resource-efficient and sustainable economy are impacting the financial sector. Climate risks (physical and transition) could negatively impact all the traditional categories of financial risks to which banking institutions are exposed. Furthermore, social factors, such as human rights, health or working conditions, and governance factors such as executive leadership or bribery and corruption can also lead to financial impacts on institutions' counterparties or invested assets and represent sources of financial risk .

The EBA (European Banking Authority) has established an "Action Plan on Sustainable Finance" (December 2019) to determine the sustainability characteristics of banking activities, where it encourages incorporating ESG issues into assessments, using, among other things, GAR (Green Asset Ratio) indicator, which represents the percentage of "green" loans in the bank's balance sheets, as well as the inclusion of ESG risks within the RAF (Risk Appetite Framework). On 18 April, the European Banking Authority (EBA) itself concluded a public consultation on recent directives for the management of risks related to the environment, society and governance (ESG). In reference to the new guidelines (hereinafter GL) issued by the banking authority itself, banks will have to be able to

[1] The thoughts and information expressed herein are solely those of the author and do not in any way bind the institution he belongs to.
[2] Bank of Italy (2024), Was Covid-19 a wake-up call on climate risks? Evidence from the greenium; Economic and Financial Issues 832, March

adequately identify, measure, manage and monitor ESG risks through robust data processing processes and a combination of methodologies. In particular, financial institutions will need to adopt a robust approach that can mitigate ESG risks, both in the short term and with a time horizon of at least 10 years, and apply a series of risk management tools, including dialogue with counterparties. Furthermore, banks themselves will also need to monitor ESG risks through effective internal reporting frameworks and a set of retrospective and forward-looking ESG risk metrics and indicators. This is necessarily with a view to guaranteeing the security and solidity of institutions in the short, medium and long term.

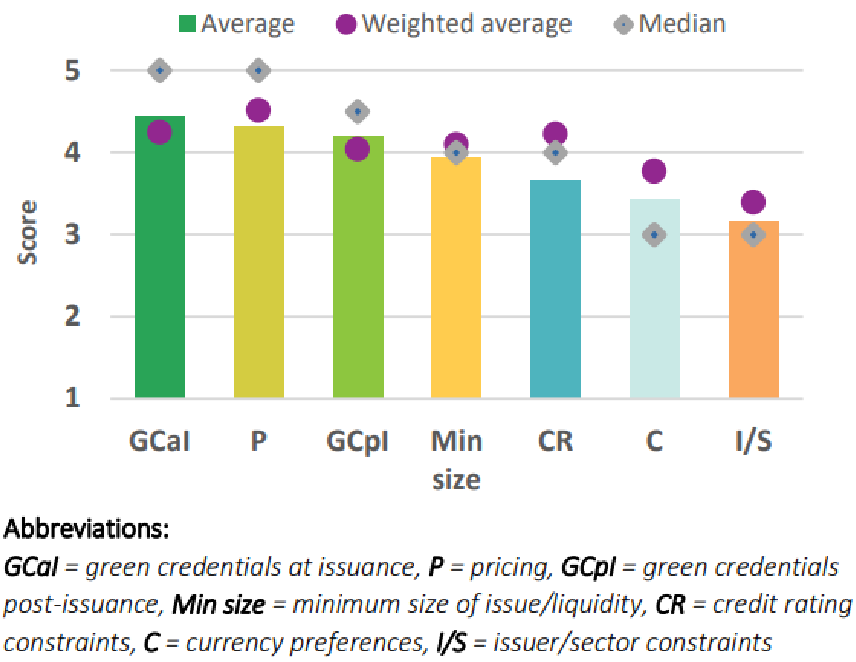
ESG risk management methodologies and processes will need to be integrated into existing frameworks, consistent with overall business and risk strategies. According to the GLs, the risk management and mitigation tools deemed necessary are:
• engagement with counterparties aimed at improving their ESG risk profile by focusing on the most important counterparties;
• adapt the contractual terms and conditions and/or, where appropriate, the pricing based on the exposure of ESG risks and the risk strategy;
• integrate ESG risks into risk limits;
• diversify portfolios based on relevant ESG criteria (sector, geographic area, etc.) and reallocate them towards exposures with a better ESG risk profile.

In addressing ESG risk, banks will need to use a holistic approach, integrating it into the current processes and metrics used to manage individual risk profiles. ESG risk, in fact, does not represent a 'stand-alone' type of risk, but exerts an influence on the financial and non-financial risks present in a bank at various levels. Therefore, risk management methods and processes need to be modified, considering the complex cause-effect relationships between risk types. This involves risk measurement and assessment techniques in run-the-bank and change-the-bank processes, as well as in stress testing applications. In addition to integrating ESG factors into the risk management framework, banks must consider related issues in product design, pricing and business strategies/decisions. From this perspective, in fact, adequate consideration of ESG risks within a wide range of change processes is of vital importance to promote profitability.

3. Use of Sustainable Bonds

ESG bonds are increasingly at the center of investing globally and GSS+ (green, social and sustainability-linked) debt markets are growing rapidly. Sustainability has become an integral part of many areas of daily life. Interest in sustainable investments comes from both institutional investors and retail savers. The risk profile of green bonds is positively influenced by the fact that the issuers are often innovative companies with more developed environmental policies and therefore less exposed to ESG risks. Decisions to invest in sustainable bonds are dictated by many factors (figure 1): on the one hand, the characteristics of actual environmental sustainability and, on the other hand, the financial characteristics, in particular the pricing of the bond (P), the size of the issue and its consequent presumed liquidity, the rating assigned to the bond (CR), the denomination currency (C), the type of issuer and the economic sector to which it belongs (I/S). Based on the survey carried out by the Climate Bonds Initiative (CBI 2019) among European asset managers, the weight of the 'green' and financial aspects tends to be equivalent overall and this is consistent with the investor's search for a balance between sustainability objectives and return and risk objectives. It is significant that larger investors tend to attribute greater importance than others to financial it is meaningful and, conversely, those with smaller assets under management look more at sustainability criteria.

Figure 1- Choice criteria for purchasing green



Source: CBI (2019)

There are four main categories of sustainable bonds to support socio-environmental initiatives:

- Green Bond (creation of a single sustainability project);

- Social Bonds (linked to new projects/refinancing existing projects with positive social impact);

- Sustainability Bond (projects that pursue both social and environmental goals);

- Sustainability-Linked Bond (projects linked to the achievement of specific sustainability objectives).

Investors' interest in sustainable bonds in recent years has been very high. One of the reasons for this interest is related to the expectation of better economic results due to lower energy consumption, lower environmental risks and the greater resilience of entrepreneurial activities in the medium-long term: the investor perceives a reduced asset risk. The results of various researchs confirm this empirical evidence (one among all Fride, G., T. Busch and A. Bassen, 2015).

Sustainable bonds are subject to the financial risks typical of the conventional bond instrument through an evaluation of its financial characteristics, in particular, the yield in relation to the duration of the instrument and the riskiness of the issuer's riskiness.

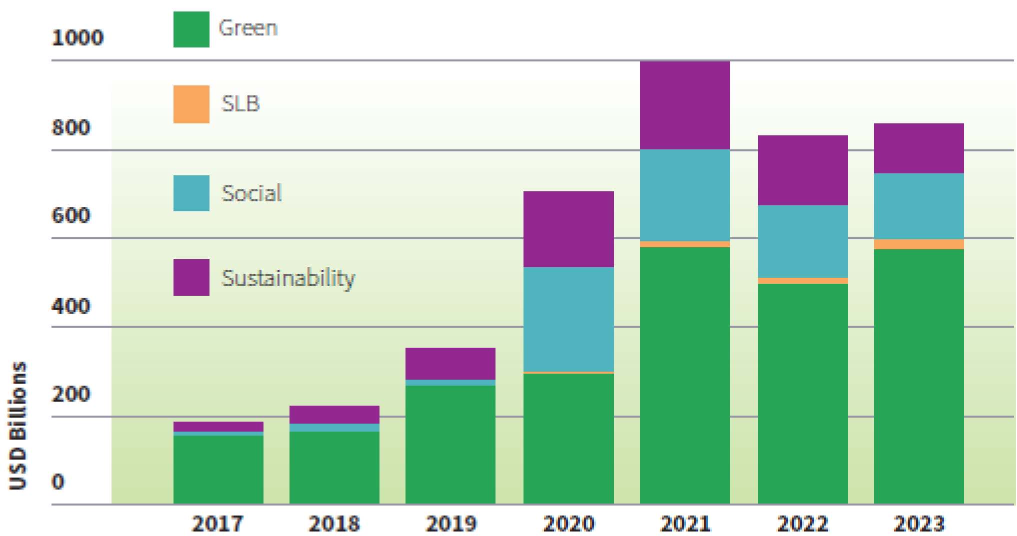
In 2023 (figure 2), Sustainable Bonds reached a volume equal to approximately 871.6 dollars, 3% more than the 2022 figure equal to approximately 842.8 billion dollars. Of these, two thirds (67.5%) are represented by green bonds which reached a volume equal to approximately $587.6 billion reflecting a 15% year-over-year increase. Conversely, there was a decline in annual volumes of Social and Sustainable Bonds of 7% and 30%. The SLB (Sustainability-linked bond) segment recorded a notable 95% increase in volumes reaching $22.9 billion in 2023 compared to $11.7 billion in 2022.

In 2023, green bond issuance saw an increase compared to the previous year.

The issuance of sustainable bonds will reach a value of 1,000 billion dollars in 2024, growing slightly compared to 2023[3].

ESG bonds are increasingly at the center of investing globally. According to European Commission estimates, around 600 billion emissions will be needed until 2030 to finance the sustainable transition. The largest green issue was launched by the Italian government in 2023 with an operation value of 10 billion euros (Treasury source).

Figure 2- Sustainable bond issues



Source: Climate Bonds[4]

The financial literature identifies three reasons underlying the motivations of companies to use this form of financing:

i. The Signaling effect: it gives the issuer a better reputation and, consequently, greater appreciation by the market. Companies that pay more attention to sustainability issues are considered by the market to be less risky and more profitable as they are more attentive to the efficiency of production processes and more open to innovation.

ii. Green washing: the possibility, in the absence of specific regulation, to deceive the market by obtaining a reputational benefit but without a concrete environmental objective. The existence of a signaling effect, referred to in point i) above, explains why some companies may be tempted to pass themselves off as "green" when they are not. In the absence of specific regulation, the attribution of the adjective "green" to the bond is essentially linked to the issuer's declaration regarding the use of the proceeds of the bond. A check-of the actual commitment in terms of sustainability can be carried out through the analysis of the environmental rating attributed to companies that issue green bonds. Studies have verified that there is an increase in this rating post-issuance of a green bond also associated with a significant reduction in CO_2 but only for issuers who have obtained certification from an independent third party.

iii. Cost of capital: the opportunity to raise funds at a lower cost, given the market's willingness to purchase these securities at a premium to comparable non-green bonds. The more sustainable companies have lower cash flow volatility and greater protection from systematic risks, thus justifying the existence of a discount on the yield of the debt issued. In a Capital Asset Pricing Model

[3] Sustainable Bond Issuance To Approach $1 Trillion In 2024, S&P Global Ratings.
[4] CBI (2023) , Global State of the Market Report , Sustainable Debt Global State of the Market

(CAPM) model (Ruefli et al., 1999), a company's beta has two important functions. First, beta measures companies' exposure to systematic risk (i.e., a lower beta means less systematic risk) and, second, it translates the equity risk premium into the required rate of return for the individual company. Therefore, lower systematic risk means that a company's equity has a lower beta value, and therefore investors demand a lower rate of return. This translates into a lower cost of capital for that company. Finally, a lower cost of capital leads directly to the last stage of the transmission mechanism: in a DCF model, a company with a lower cost of capital also enjoys a higher valuation.

The other reasons towards green bonds are to be found in the new needs of financial intermediaries who have strategies linked to the decarbonisation and sustainability of their own investments and those of their customers; on the other hand in the new indications of the regulators who have dictated stringent rules to be respected. Last but not least, the Bank of Italy, which has precisely indicated what are the risks that are linked to global warming, in particular for credit activities ("Supervision expectations on climate and environmental risks").

In February 2023, the European Union reached a provisional agreement on the creation setting up of a European standard, called the European Green Bond Standard (EGBS). According to this agreement, all proceeds from the EuGB will have to be invested in economic activities aligned with the EU taxonomy. For sectors not yet covered by the EU taxonomy and for some very specific activities, there will be a flexibility of 15%, in order to ensure the usability of the European green bond standard from the beginning of its existence. Subsequently, to support the growth of green bonds and promote the transition, the European Union (2023) adopted a regulation that will come into force on 21 December 2024 (EU Regulation 2023/2631) known as the European Green Bond Standard (EuGb). Green bonds are useful tools for financing investments in green technologies, energy and resource efficiency, as well as green transport infrastructure and research-focused infrastructure. Issuers will be able to demonstrate that they finance green projects in line with the EU taxonomy. Investor confidence in green investments will be strengthened thanks to a framework that reduces the risks posed by greenwashing, ultimately stimulating capital flows into environmentally sustainable projects. To avoid greenwashing in the green bond market, the regulation also includes some voluntary disclosure requirements for other green bonds and sustainability-related bonds issued in the EU. The proceeds from European green bonds must be used to finance economic activities that have a lasting positive impact on the environment (those identified as sustainable by the taxonomy regulation 2020/852/EU). Until the taxonomy is fully operational, issuers of an EU certified green bond must ensure that at least 85% of the funds raised by the bond are allocated to "sustainable" economic activities. The remaining 15% can be allocated to other economic activities, provided that the consumer information rules are respected.

In the near future, the supply of green bonds will likely be supported by investor demand. The European green bond standard (Eugbs) will be able to contribute to the growth of this demand, provided that large public and non-public actors (national governments, banks) follow the standards introduced by the regulation, giving credibility to the entire system.

4. Greenium and the factors that influence it

Green bonds are the dominant type of issue in the sustainable bond market. They are characterized by a greenium, i.e. a premium price compared to comparable brown ones, consistent with the theory of investor preference and the incorporation of the protection offered by environmental risk factors into the bond price. For corporate bonds, the market recognizes the discount only in the presence of a certification issued by a third-party and independent body regarding the greenness of the issue.

The presence of greenium can be verified by taking both the primary and secondary markets as a reference (tab. 1). In the first case, greenium consists of a lower coupon or a higher issue price compared to conventional securities with similar characteristics, which leads to a lower return for investors in sustainable instruments and a lower cost of money for investors. green broadcasters. In the secondary market, the presence of greenium takes the form of lower spreads (calculated with reference to a risk-free rate curve) compared to conventional bonds of the same duration, rating and issuer. The lower financial return could in fact be justified by the search for non-financial objectives linked to environmental sustainability aspects.

Table. 1- Results of empirical studies on greenium

authors	Period under observation	Market segment	Objectives of the study	Main results
Baker et al (2018	2010-2016	Primary market	Existence of greenium on US local government green bond issuances versus brown counterparts	US local government green bonds are issued at a negative premium of approximately 7 basis points over similar conventional bonds.
Hachenberg, Schiereck (2018)	October 2015- March 2016	Secondary market	Existence of price differentials (daily i-spread) between a sample of green bonds and a comparable sample of brown bonds	On average, the differences in yield between green and brown bonds are limited (- 1 bp); they are more evident for the A rating class while AAA ones discount positive premiums compared to non-green bonds.
Karpf, Mandel (2018)	2010-2016	Secondary market	Comparison of yield curves of bonds issued by US local governments, both brown and green	The yield curves of green bonds are located below the respective brown bond curves, with yield differences increasing as time increases. However, after taking into account the credit quality of the issuer and the technical characteristics of the two bond classes, green bonds show a positive premium of around 7bps

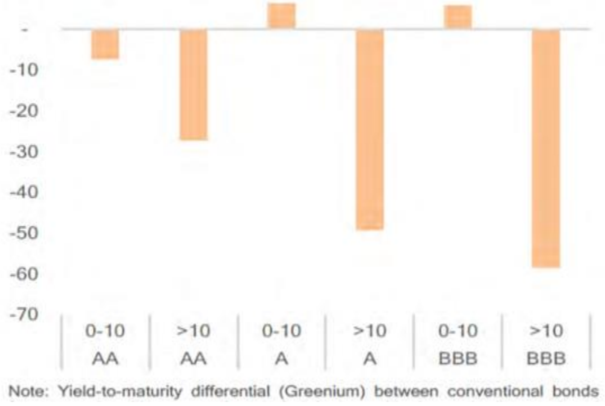
CBI (2019)	2019	Primary market	Identify the existence of an issue spread with respect to the yield curve of outstanding securities (both brown and green) as it happens for brown securities	2/3 of green issues are placed on the yield curve of the securities in circulation; the green bonds that benefit from a discount at issuance are referable to supranational issuer
Fatica e al. (2019)	2007-2018	Primary market	Determinants of the yield at issuance	Only supranational issuers (with 80 bps) and corporates (20 bps) benefit from the greenium; not so the issues of financial companies. Being a "serial" issuer of GB leads to greater savings
Larcker, Watts (2019	2013-2018	Primary market	Comparison of 640 Green and non-Green issues with the same issue date, maturity, rating and issuer (US local authority)	Trivial difference in the required yield (0.45 bps). In 85% of cases the difference is zero. There is no greenium and this does not depend on green washing problems
Zerbib (2019)	2013-2018	Primary market	Comparison between over 1000 green issues and a sample of "synthetic" bonds with the same characteristics net of the "green" label and a different degree of liquidity, which is specifically controlled	Greenium present but limited, about -2 bps for the entire sample. The negative difference is more pronounced for financial institutions issues and for bonds of lower credit quality
Gatti, Florio (2020)	2007- 2015	Primary market	Determinants of the Issue Spread, distinguishing between bonds with second party review and bonds applying GBP	Issue spreads have increased following the introduction of GBP, but at the same time the number of bonds issued with medium/low ratings has increased (which could explain the unexpected increase); having a second party review allows for savings in terms of issue spreads.

Source: Monetary Observatory (2020)

An investment in sustainable activities and projects by companies would have positive effects in terms of fewer environmental and social risks, including legal and reputational ones.

In a recent study carried out by ESMA, the actual yields of green bonds with traditional ones were compared both by rating class and by maturity (figure 3). The graph below shows that the existence of a discount on the effective yield exists and is positively correlated with the maturity of the bond, with particular reference to the long term and negatively correlated with the issuer's rating.

Figure 3 – Green vs conventional bond yields - Grenium concentrated in longer maturities



Source: Esma

Note: Yield to Maturity differential (Greenium) between conventional bonds from green bond issuers and green bond, by residual maturuty (in years) and creting rating in bps. Data as at November 2021

Bank of Italy has carried out an empirical analysis on the existence of the so-called greenium. Furthermore, the hypothesis was tested that Covid-19 has increased attention (wake-up call) towards the risks linked to extreme shocks, such as those potentially induced by climate change. The results show the presence of a greenium on a sample of international bonds for the period 2017-2022, also attributable to the strong demand for green bonds from investors. The pandemic shock acted as a wake-up call for climate risks only temporarily, resulting in an expansion of greenium that disappeared after the emergency. In this context, in order to be able to analyze the supply and demand of green bonds during the covid and post-covid period, the Bank of Italy used a methodology based on a disequilibrium model (Fair and Jaee (1972) and Maddala and Nelson (1974)) to estimate supply and demand for green bonds. Unlike an equilibrium model where the compensation rule is implemented, in the disequilibrium model the price of securities is an exogenous variable and possible misalignments between supply and demand could lead to an aggregate excess of demand or supply. By applying an imbalance model in the analysis carried out, an excess of demand may be seen on the secondary market for green bonds.

Below are the equations used to estimate QS demand and QD supply:

1) $Q^{s}{}_{i,h,s,t} = f^{s}(X^{s}{}_{i,h,t},\ X^{s,D}{}_{i,h,s,t}) + \varepsilon^{s}{}_{i,h,s,t}$

2) $Q^{D}{}_{i,h,s,t} = f^{D}(X^{D}{}_{i,h,t},\ X^{s,D}{}_{i,h,s,t}) + \varepsilon^{D}{}_{i,h,s,t}$

Equation 1 and 2 indicate how supply and demand both depend on specific variables. In particular, $X^{s}i,h,t$ are supply-specific factors just as $X^{D}{}_{i,h,t}$ are demand-specific factors.

The equation below indicates the observed quantity of the green bond as the lesser of the quantity demanded and the quantity supplied.

3) $Q_{i,h,s,t} = min(Q^{S}{}_{i,h,s,t},\ Q^{D}{}_{i,h,s,t})$

With reference to the supply and demand shocks reported above, $\varepsilon^{s}{}_{i,h,s,t}$, $\varepsilon^{D}{}_{i,h,s,t}$ (equation 1.2) it is assumed that the latter are not correlated.

Table 2 reports the descriptive statistics of the observed sample (2017-22 period) of green bond

Table 2- Descriptive statistics of Green Bonds

Year	Variable	N	Mean	SD	Median	Min	Max
2017	(log) Quantity	5,417	1.01	2.19	1.09	-7.,10	9.26
	(log) Price	5,417	0.32	1.14	0.21	-2.06	6.50
	(log) Amount	5,417	6.54	0.97	6.22	-0.18	10.17
	Residual maturity	5,417	7.73	2.82	7.22	3.76	19.88
	Rating	5,417	16.65	3.16	16.00	7.50	21.00
2018	(log) Quantity	8,856	0.92	2.15	1.00	-8.27	6.81
	(log) Price	8,856	-0.07	1.10	-0.21	-2.25	5.22
	(log) Amount	8,856	6.35	1.00	6.21	-13.95	8.23
	Residual maturity	8,856	6.88	2.99	6.36	0.12	19.60
	Rating	8,856	16.34	3.32	15.67	6.50	21.00
2019	(log) Quantity	13,170	0.94	2.13	1.03	-8.65	8.94
	(log) Price	13,170	0.55	1.19	0.60	-2.21	5.32
	(log) Amount	13,170	6.42	0.67	6.35	-0.12	9.94
	Residual maturity	13,170	6.97	3.46	6.39	0.95	19.90
	Rating	13,170	15.66	3.26	15.00	6.50	21.00
2020	(log) Quantity	14,367	0.78	2.16	0.80	-10.33	9.11
	(log) Price	14,367	0.95	1.49	0.84	-2.30	6.50
	(log) Amount	14,367	6.34	0.74	6.35	-0.20	10.22
	Residual maturity	14,367	7.03	3.78	6.50	0.79	19.93
	Rating	14,367	14.91	3.32	14.33	5.00	21.00
2021	(log) Quantity	29,936	0.88	2.15	0.95	-13.62	9.03
	(log) Price	29,936	0.42	1.24	0.35	-2.26	5.55
	(log) Amount	29,936	6.42	0.73	6.27	-1.73	10.34
	Residual maturity	29,936	6.64	3.61	6.07	0.03	19.95
	Rating	29,936	15.11	3.35	14.33	6.00	21.00
2022	(log) Quantity	36,133	0.66	2.21	0.71	-14.49	9.11
	(log) Price	36,133	-1.38	0.30	-1.34	-2.30	1.25
	(log) Amount	36,133	6.59	0.87	6.40	-0.06	10.42
	Residual maturity	36,133	5.52	3.78	4.77	0.03	19.98
	Rating	36,133	15.11	3.35	15	6.21	14.32

Source: Bank of Italy (2024)

Table 3 shows the estimates obtained using the disequilibrium model. We consider the logarithmic values for both the quantities (AMOUNT) and the prices (PRICE) of green securities. Without considering the pandemic effect generated by Covid-19, on the supply side (Column 3) we can observe a positive and statistically significant coefficient associated with both the price (PRICE) and the amount issued (AMOUNT); similarly, the residual life and ratings of the securities also have a positive and significant impact on the supply side of green securities. On the demand side, a positive and statistically significant coefficient can be seen on both the total amount issued and the rating. Furthermore, non-linear effects on the residual life of the securities can be seen while the coefficient associated with the price appears to be statistically insignificant. With the outbreak of the pandemic, the results for the demand equation (Column 2) provide evidence of the usual relationship between prices and quantities: the coefficient associated with PRICE is always negative and statistically significant on both the demand and supply side). Furthermore, the coefficient associated with the interaction between COVID and PRICE is significant and positive for both sides of the market (table 3, columns 2 and 4). This may depend on whether investor preferences for sustainable assets can be adequately incorporated into prices. After the Covid-19 shock, investors are willing to buy green stocks at the price set before the pandemic or to maintain the same orders in case of a price increase. Overall, the minimum quantity (equation 3) is always driven by demand over the entire sample period (figure 4).

Table 3 - Results Demand and supply

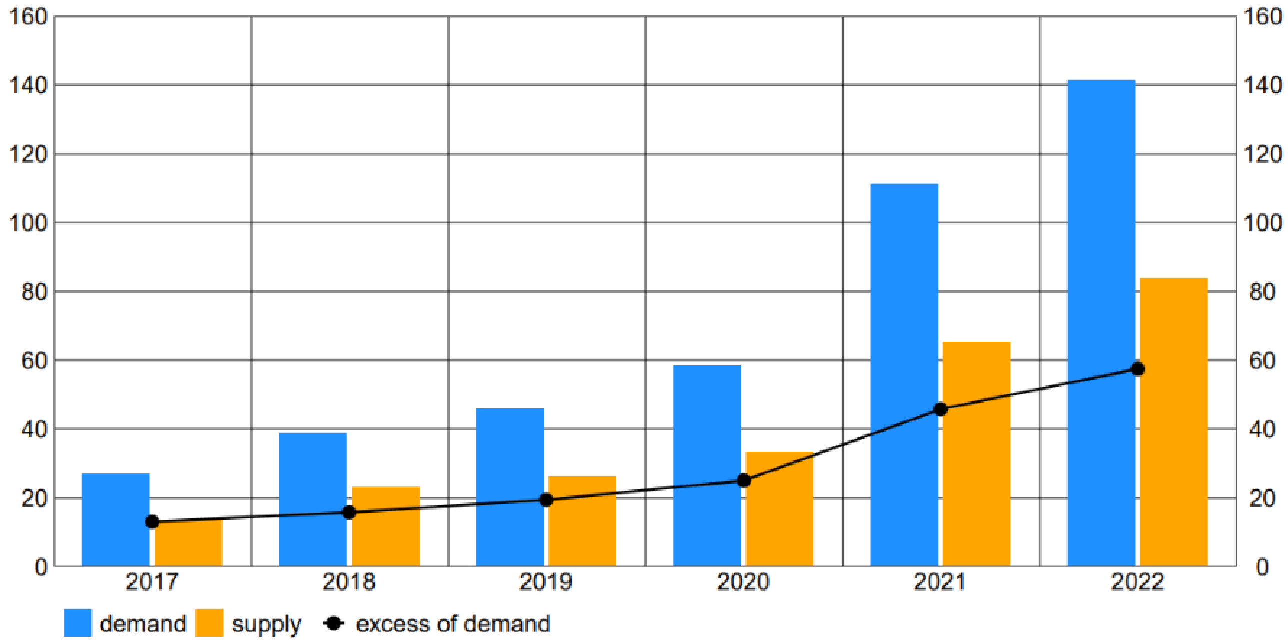
QUANTITY	DEMAND EQUATION COVID		SUPPLY EQUATION COVID	
	NO	YES	NO	YES
PRICE	-0.0042	-0.0326***	0.0330***	-0.0125
	(0.0046)	(0.0078)	(0.0058)	(0.0096)
COVID		0.4225		-0.0186
		(0.7703)		(0.4913)
COVID x PRICE		0.0392***		0.0630***
		(0.0086)		(0.0106)
AMOUNT	0.2105***	0.2102***	0.0974***	0.0970***
	(0.0038)	(0.0038)	(0.0048)	(0.0048)
MATURITY	-0.0097	-0.0062	0.0317***	0.0374***
	(0.0074)	(0.0075)	(0.092)	(0.093)
MATURITY^2	-0.0023**	-0.0027***	-0.0015	-0.0022*
	(0.0009)	(0.0009)	(0.0011)	(0.0011)
MATURITY^3	0.0001***	0.0002***	0.0001	0.0001**
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
RATING^2	0.0010***	0.0010***	0.0023***	0.0021***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Issuer Country x Time FE	Yes	Yes	Yes	Yes
Issuer Sector x Time FE	Yes	Yes	Yes	Yes
Currency x Time FE	Yes	Yes	Yes	Yes
Holder Country x Time FE	Yes	Yes	No	No
Holder Sector x Time FE	Yes	Yes	No	No
Lagged share by Holder Country	No	No	Yes	Yes
Lagged share by Holder Sector	No	No	Yes	Yes
N	300,552	300,552	300,552	300,552
R^2	0.403	0.403	0.403	0.403

Standard errors in parentheses
* $p < .10$, ** $p < .05$, *** $p < .01$

Source: Bank of Italy (2024)

Excess demand increased slightly after Covid (figure 4) and increased further during 2021, while a deceleration was observed at the end of 2022.

Figure 4 - Predicting excess demand (euro billions)



Source: Bank of Italy (2024)

Green bonds are on average more liquid than traditional bonds. In order to be able to estimate the yield of green securities that takes into account both the different ratings and the stock market listing, a linear fixed effects model was used to estimate the Yield to maturity (Y,s,t) in which the dependent variable is represented by the same yield to maturity measured at the end of month t.

The explanatory variables used in the regression are: the residual duration of the security (MATURITY), the LISTED variable equal to one if the security is listed on the stock exchange, the RATING, the amount (AMOUNT), the GREEN variable and the pandemic variable out of three levels: equal to zero before March 2020, equal to one between March 2020 and March 2022 (COVID) and equal to two for the period after the first quarter of 2022 (POST COVID).

The fixed effects regression model used is shown below:

$$Y_{s,t} = Y_e GREEN_{s,t} + Y_c COVID_t + Y_p POSTCOVID_t + \boldsymbol{\theta}^T X_{s,t} + \eta_{i,t} + \eta_{e,t} + \eta_{u,t} + \varepsilon_{s,t}$$

The analysis is carried out on three sectors: financial, non-financial and public. From the results obtained, a lower return can be seen, equal to 3-14 basis points depending on the sector to which it belongs (table below).

A higher rating results in a lower return as it incorporates a lower risk premium, furthermore the higher the amount issued, the lower the return; this depends on higher volumes and a greater number of investors.

The coefficients of the indicator variable Green are all statistically meaningful in the three sectors of the basic model. The greenium, estimated at 5 basis points for non-financial companies, is even higher (14 basis points) for financial companies, while it is lower for the public sector (3 basis points). Next, it was observed whether the Covid-19 shock had an effect on the return.

From Tab. 4, it can be seen that the pandemic (GREEN x COVID). led to a further negative premium on green bonds issued by non-financial and financial operators (5 basis points) while there is no evidence of a further negative premium on those issued by the government sector.

However, after the end of the state of emergency, a reduction in negative greenium of 6 and 7 basis points is found, compared to the pre-pandemic period. Greenium's post-pandemic rebound indicates that investors temporarily factored in climate risks, resulting in a greenium expansion that lapsed post-emergency.

Table 4- Regression results

	Corporations Non-financial	Corporations Financial	Government	Corporations Non-financial	Corporations Financial	Government
	(1)	(2)	(3)	(4)	(5)	(6)
GREEN	-0.0475*** (0.0064)	-0.1391*** (0.0071)	-0.0298** (0.0133)	-0.0387*** (0.0137)	-0.1354*** (0.0149)	-0.0251 (0.0247)
GREEN x COVID				-0.0504*** (0.0163)	-0.0540*** (0.0182)	-0.0092 (0.0319)
GREEN x POST COVID				0.0641*** (0.0184)	0.0699*** (0.0197)	-0.0028 (0.0350)
LISTED	-0.0349*** (0.0030)	-0.0538*** (0.0040)	0.0074 (0.0061)	-0.0349*** (0.0030)	-0.0539*** (0.0040)	0.0074 (0.0061)
RATING	-0.3294*** (0.0026)	-0.2191*** (0.0008)	-0.2467*** (0.0022)	-0.3293*** (0.0026)	-0.2191*** (0.0008)	-0.2467*** (0.0022)
AMOUNT	-0.0251*** (0.0006)	-0.0844*** (0.0011)	-0.0100*** (0.0005)	-0.0250*** (0.0006)	-0.0844*** (0.0011)	-0.0100*** (0.0005)
MATURITY	0.2841*** (0.0019)	0.2881*** (0.0025)	0.1909*** (0.0041)	0.2840*** (0.0019)	0.2880*** (0.0025)	0.1909*** (0.0041)
$MATURITY^2$	-0.0128*** (0.0002)	-0.0140*** (0.0003)	-0.0048*** (0.0005)	-0.0128*** (0.0002)	-0.0140*** (0.0003)	-0.0048*** (0.0005)
$MATURITY^3$	0.0002*** (0.0000)	0.0003*** (0.0000)	-0.0000 (0.0000)	0.0002*** (0.0000)	0.0003*** (0.0000)	-0.0000 (0.0000)
Issuer x Time FE	Yes	Yes	No	Yes	Yes	No
Country x Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Currency x Time FE	Yes	Yes	Yes	Yes	Yes	Yes
N	430,146	456,655	133,024	430146	456,655	133,024
Adjusted R^2	0.939	0.862	0.908	0.939	0.862	0.908

Source: Bank of Italy (2024)

Conclusions

Over the years, ESG factors are playing a important role in the investment decision-making process. Sustainable finance is defined as the incorporation of ESG (environmental, social and governance) factors into investment and financing decisions with the aim of obtaining long-term returns and contributing to sustainable development.

This approach goes beyond simple economic profit because it seeks to generate a positive impact on society and the environment. Governments and supranational institutions play a key role in the growth and development of this new type of finance, they must try to encourage this type of investment to deal with all the environmental and social problems that are arising in recent years.

Important regulatory work will be necessary, new laws will be needed (clear, simple, but at the same time complete), to give the investor all the information he needs and also support throughout the entire life of the operation. In this context, in the world of sustainable finance, green bonds with a similar functioning to traditional bonds play a fundamental role due to the fact that the amount raised will be exclusively used to finance environmental projects.

Investors are willing to give up a small part of the return compared to traditional securities because they are rewarded by the positive environmental impact. Issuers, on the other hand, can take advantage of a slightly lower cost of debt provided they finance projects with specific purposes.

A further acceleration of this phenomenon is foreseeable in the immediate future, since many banks, also encouraged by the Prudential Supervision discipline, are planning to issue green finance products in which green bonds will be a very important lever.

The global bond market will be an important source of investment to drive the climate transition. The new European Standard (EuGB) to regulate green bonds and combat greenwashing will make investors' choices more transparent and safer and will favor those companies that are able to show the effectiveness of ongoing projects with the support of methodologies and data.

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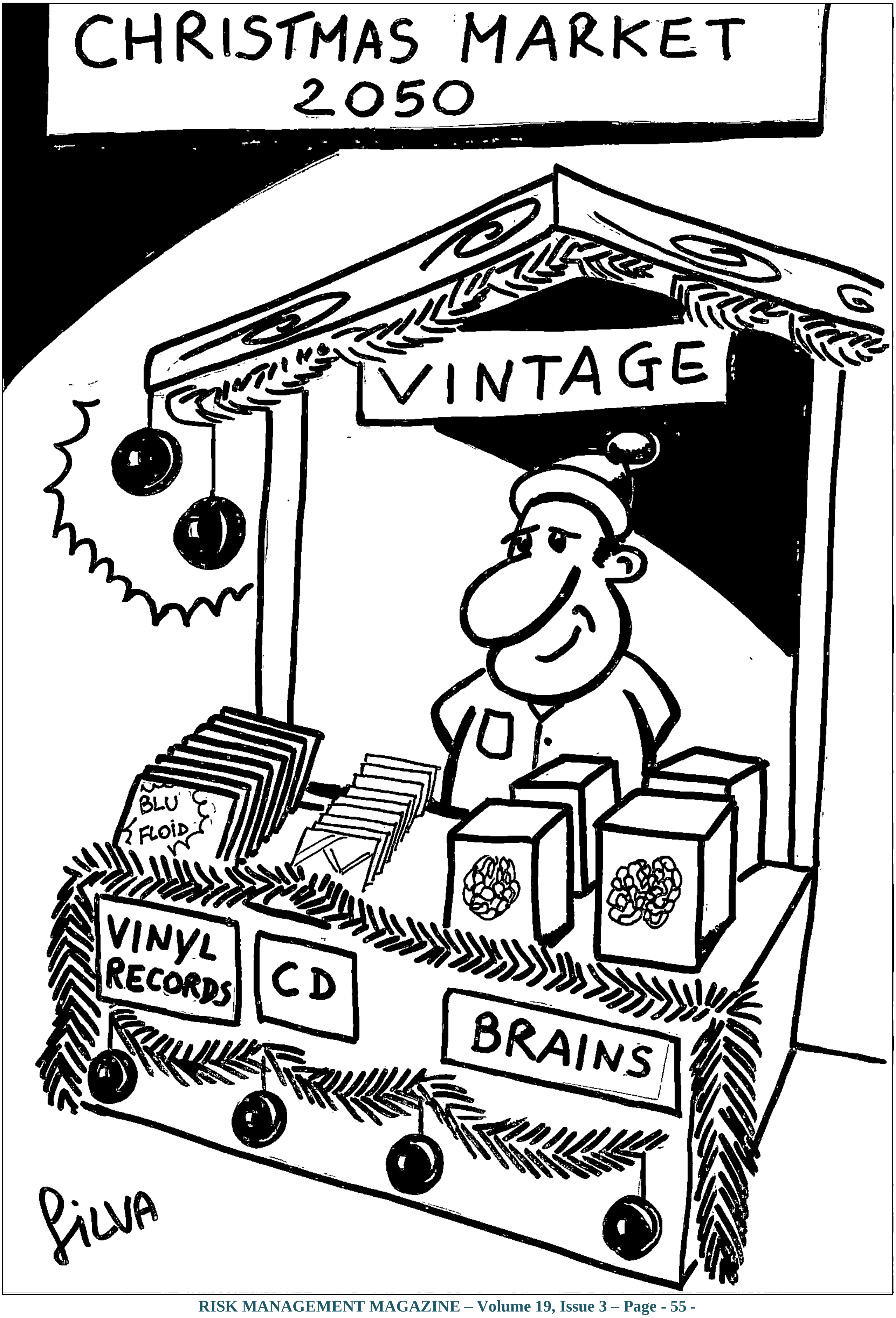
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